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ANALYSIS OF THE "JOGGLE-LAP" JOINT FOR AUTOMOTIVE APPLICATIONS

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Analysis of the "Joggle-Lap" Joint

for

Automotive Applications

Richard C. Givler

R. Byron Pipes, Thesis Advisor

submitted as a Senior Thesis in partial fulfillment for a Degree with Distinction

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Abstract

An analytical model is developed to describe the response of the "joggle-lap" joint to both tensile and bending loads. The model consists of a non-linear beam analysis which calculates stress profiles through the adherent thickness. A plane stress finite-element model was incorporated into the analysis to correctly determine the stress field in the adhesive zone where it was shown that beam analysis was less accurate. Elastic response of the "joggle-lap" joint due to tensile loads was verified through experimental testing and ultimate loads were accurately predicted within experimental error. Maximum adherent flexural stress was found to determine joint failure. A parametric study was undertaken by using the verified analytical model and the results were recorded as a series of design curves.

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Nomenclature

a	cross-sectional area
b	adhesive bond thickness
C_0, C_1, C_2	constants
DEFLA	deflection
DUDSA	angular rotation
e	eccentricity
e_ℓ	elongation
E	modulus of elasticity for an isotropic material
E_f	modulus of elasticity of fibers
E_m	modulus of elasticity of matrix
E_n	modulus of elasticity normal to the fiber plane
E_x, E_y, E_z	modulus of elasticity of a general anisotropic body
F	applied force
F_i i=1-5	force components
h_i i=1-5	nodal points
H_i i=1-5	axial force of SEGi
I	moment of inertia about neutral surface
$I_{eq.}$	equivalent moment of inertia
ℓ_1	length of SEG1
M_i i=1-5	applied moment at SEGi
M_{corr}	correcting moment
M(S)	moment distribution

Nomenclature (Cont'd)

R	radius of curvature
$SEGi_{i=1-5}$	beam elements of the "joggle-lap" joint
S	specific gravity
S_i $i=4-6$	ultimate shear strength
t	adherent thickness
$\bar{u}$	distance between neutral axis and centroidal axis
u_o	deflection at the end of SEG1
u_i, s_i $i=1-5$	local coordinate system corresponding to individual beam element
v_f	volume fraction of fiber
v_m	volume fraction of matrix
V_i $i=1-5$	shear force on SEGi
W	weight fraction
X,Y	global coordinate system
Y	radial coordinate in curved beam members
X_i^T	ultimate tensile strength
X_i^C	ultimate compressive strength
α	angle measure
Δ	infinitesimal difference
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	strain components
ε_{ult}	ultimate strain
Θ	angle measure
λ	linear measure

Nomenclature (Cont'd)

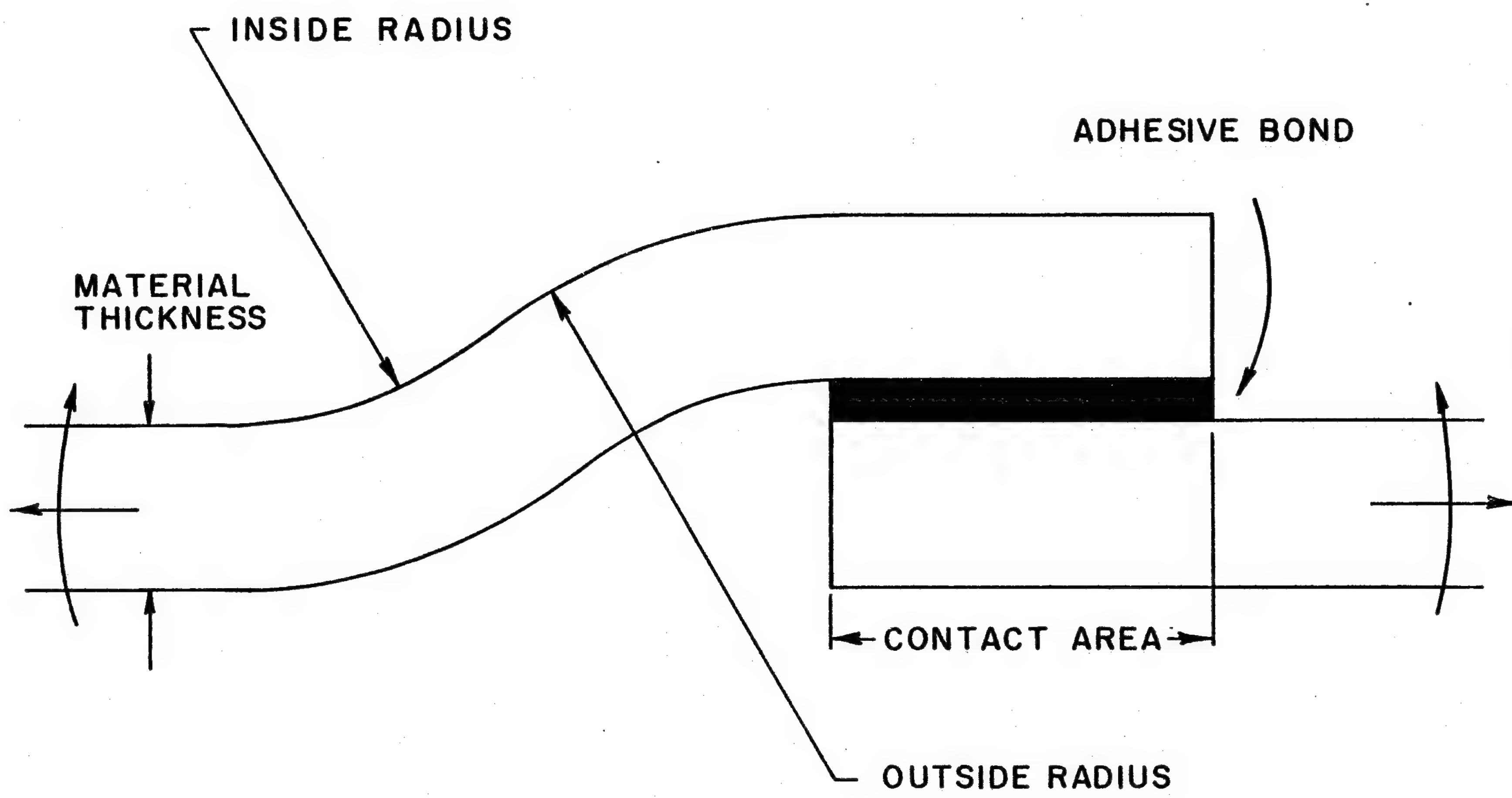
ν_{ij}	Poisson's ratio
π	3.14159...
$\sigma_1, \sigma_2, \tau_{12}$	plane stress components
$\sigma_x, \sigma_y, \sigma_{xy}$	
σ_{ult}	ultimate strength
ϕ	angle measure

I. Introduction

Recent government regulations for increased gasoline mileage requirements have induced automobile manufacturers to seek light weight replacement material systems for existing metal parts. Since the automotive industry is a high volume operation, sheet molding compound (SMC) parts offer a feasible answer to the problem. The SMC molding time of from 1 to 3 min/piece depending on the size and thickness of the part is compatible with automotive assembly line production.

International Harvester et al are currently employing SMC molded body components on their vehicles to replace former sheet metal parts. This new direction has brought with it several problems, one of which is the design of adhesive joints. The joint must accommodate high rate fabrication techniques and provide optimum strength and durability. In addition, the joint must satisfy certain cosmetic requirements such as adjacent flush edges. With these criteria in mind, the "joggle-lap" joint has been chosen for detailed study and analysis. This joint configuration is shown in Figure 1. Since a joint of this type experiences a variety of loading conditions in practice, it was decided to model the joint

FIGURE 1: THE JOGGLE LAP JOINT SUBJECT TO TENSILE AND BENDING LOADS



in pure tension and pure bending. By superposition, it is apparent that any combination of these two loading conditions may then be constructed.

This work focuses on the development of an analytical model to describe the behavior of the "joggle-lap" joint due to both tensile and bending loading conditions. The first section utilizes small deflection beam theory for both straight and curved beam elements to obtain a solution for the displacement and stress fields of the joint. Included in this analysis is the derivation of the governing differential equations for the deflection of the curved beam.

The second section utilizes a finite-element model to reveal localized stress concentrations in the adhesive zone. Boundary conditions for the finite element model are obtained from a transformation of stresses in the deformed geometry to equivalent stresses in the undeformed geometry. This transformation of stresses is performed via a computer routine for ease of calculation.

Finally, experimental verification of the analytical predictions is reported along with a description of testing procedures. The maximum flexural stress is shown to correlate strength data and failure analysis. Also, the microstructure of the joint was examined as a possible explanation of the failure mode.

II. Background

A. Adherent Materials

The adherents of the proposed "joggle-lap" joint were composed of a random-fiber composite known as SMC-25. SMC is defined as a sheet molding compound that contains reinforcements with an average fiber length of approximately 1 inch (2.54 cm) with random orientation in the plane. The number 25 indicates that the composite is 25 percent glass fibers by weight. The major constituents of SMC are E-glass fibers and a styrenated polyester resin in the form of a paste. It is quite common to use mineral fillers during the manufacture of the paste to facilitate flow when molding or to obtain certain characteristics from the molded part such as a high resistance to flame or increased stiffness. Another prime reason for using fillers is the fact that they are much cheaper than the polyester resin itself and thus reduce the cost of materials. At times, chemical additives may also be introduced into the paste to serve as catalysts during the molding cycle.

The process of SMC manufacturing is a highly innovative one which is completely automated. Figure 2 (taken from Owens/Corning Fiberglass SMC Review) depicts

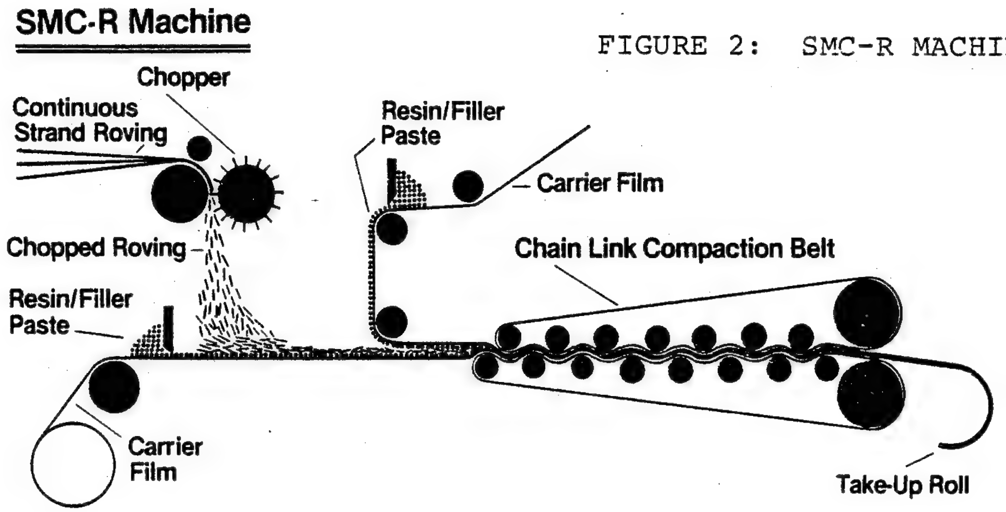


FIGURE 2: SMC-R MACHINE

a typical process currently in use by a competitive supplier of SMC. The first step of the procedure is to distribute the resin onto a polyethylene carrier film as shown. Continuous glass fibers are then chopped into lengths of less than three inches and distributed in a random fashion on the wetted film. A second layer of resin-coated polyethylene film serves as a top layer to the sandwich-like sheet. Several rows of rollers act to insure that the glass fibers are fully impregnated with the polyester resin thus yielding consistency in moldability of the SMC. Finally the product is directed to a take-up roll for ease of handling during shipping and storage.

SMC is usually placed in a constant temperature room while storing to allow maturation to take place. Maturation is nothing more than allowing the SMC to increase

in viscosity to enhance relative ease of handling of the sheet. Maturing the SMC sheet for extended periods of time greatly reduces the flow characteristics of the product while molding. Recommended shelf-life for SMC stored at 10-15^{o} C is about 2 weeks, however in general it may often be used up to 2 months after the date of its manufacture.

Once the SMC sheet has reached maturity, it is ready for molding. Upon removing the protective polyethylene film, the molding compound is cut to size and strategically placed in the mold. This procedure is known as charging the mold. The so-called strategic locations of the mold are those positions that allow the SMC to flow to all parts of the mold and maintain uniform part thickness. To date these locations have been determined by trial and error coupled with experience.

Compression molding combines both temperature and pressure to induce an exothermic reaction which serves to cure the part in the mold. Figure 3 (taken from ref [3]) is an example of a typical curing cycle showing the temperature of the part as a function of time. It should be noted that platen temperatures of 200^{o} C are usually sufficient for SMC molding and may be achieved with superheated steam. Another important fact seen from the figure is the overall cure time. Average cure times are generally

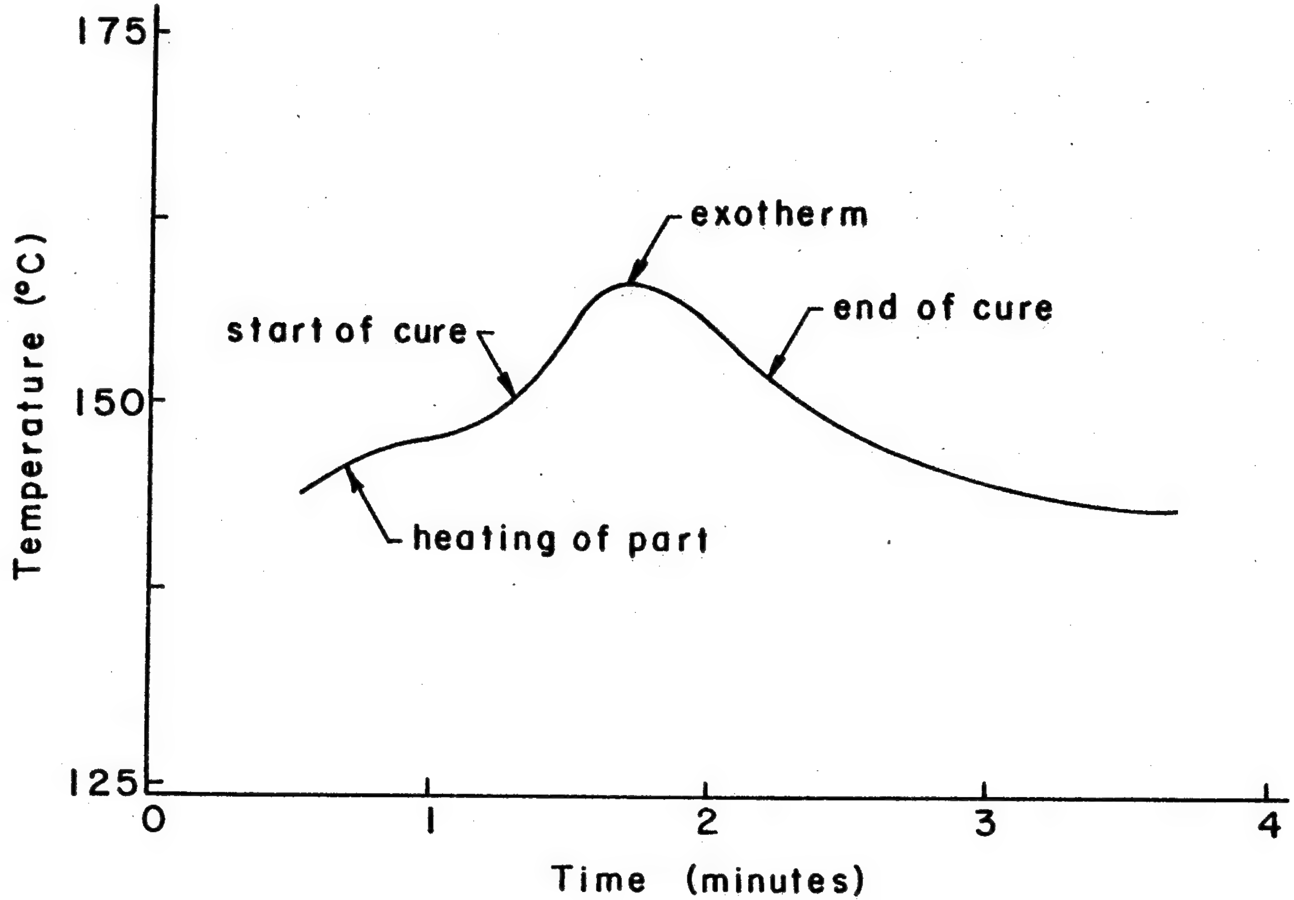


FIGURE 3: TYPICAL CURE CYCLE OF SMC

1-3 minutes (depending upon the thickness of the part) which lends itself to production line applications inherent in the automotive industry. Figure 4 (taken from ref [3]) shows the effect of pressure upon a typical cure cycle. Note that the peak pressure and maximum temperature correspond to the initiation of the exothermic reaction. The key to successful molding is to acquire fine control of the application of pressure to the cure cycle.

The main feature of SMC is the ability of the glass fibers to flow with the paste during the molding process. Since the fibers are transported to all parts of the mold, it is possible to produce a geometrically complicated part with quasi-constant mechanical properties. It has been shown by Pipes and Taggart [ref 5], that in areas of intensified flow, the fibers tend to align themselves with the direction of flow and thus produce areas of varying mechanical properties. It is therefore beneficial to understand the flow characteristics within the mold to produce a part with controlled and/or uniform mechanical properties. Taggart et al have determined the properties of SMC-25 to be those found in Table 1. Some scattering in the data was reported due to the inherent local variations in the material. To determine the normal modulus (modulus normal to the plane of the fibers), the relationship shown may be

FIGURE 4: PRESSURE VARIATION OF A TYPICAL CURE

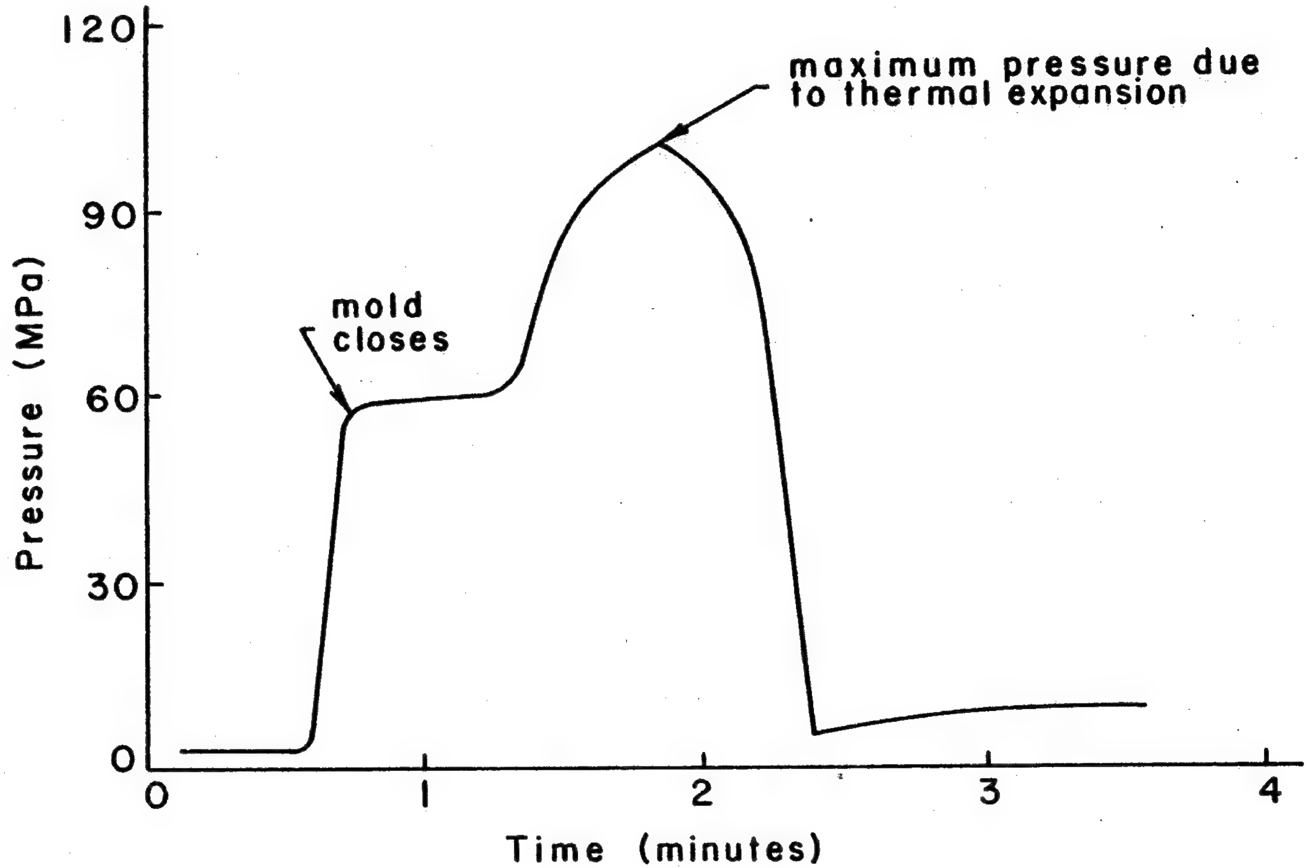


Table 1

Properties of SMC-25

Tension			
$E_L^{tension}$	GPa (Msi)	14.48	(2.1)
$\nu^{tension}$			.3
$\sigma_{ult}^{tension}$	MPa (ksi)	90	(13.1)
$\varepsilon_{ult}^{tension}$	(μ in/in)		11,400
Compression			
$E_L^{compression}$	GPa (Msi)	12.41	(1.8)
$\nu^{compression}$			.28
$\sigma_{ult}^{compression}$	MPa (ksi)	204	(29.6)
$\varepsilon_{ult}^{compression}$	(μ in/in)		20,600

used. This relationship resembles the well-known rule of mixtures for continuous fibrous composites.

$$\frac{1}{E_n} = \frac{v_f}{E_f} + \frac{v_m}{E_m} \tag{1}$$

where E_n = normal modulus of elasticity of the composite

v_f = volume fraction of fiber

v_m = volume fraction of matrix

E_f = modulus of glass fiber

E_m = modulus of matrix

Table 2 provides the needed data for determining the normal modulus of elasticity. By definition, SMC is composed

Table 2[1]

	polyester resin	E-glass fiber
Modulus of Elasticity (10^6 psi)	.5	10
Specific gravity	1.28	2.54

of 25% fiber by weight. Utilizing the equation written below

$$v_f + v_m = 1 \tag{2}$$

allows one to solve for v_f where v_m may be rewritten as

$$v_m = v_f \left[\frac{S_f}{S_m}\right] \cdot \left[\frac{W_m}{W_f}\right]$$

S_f = specific gravity of fiber

S_m = specific gravity of matrix

W_m = weight fraction of matrix

W_f = weight fraction of fiber

Making the appropriate substitutions, Eq. (2) becomes

$$\frac{.75}{.25} \left[\frac{2.54}{1.28}\right] v_f + v_f = 1$$

Thus the corresponding volume fraction of fiber and matrix are .14 and .86 respectively. From Eq. (1) the value of E_n is now calculated to be 0.58×10^6 psi.

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[1]Vinson and Chou

B. Adhesive Materials

The adhesive system chosen for the "joggle-lap" joint was developed by the Adhesives Division of Goodyear Chemicals. The Pliogrip 6000 series is a general purpose structural adhesive with a polyurethane base. Currently available as a two-part system, Pliogrip 6000 exhibits both high flexibility and resilience. With the proper selection of curatives, the working time of the adhesive may be accurately controlled between 1-6 minutes.

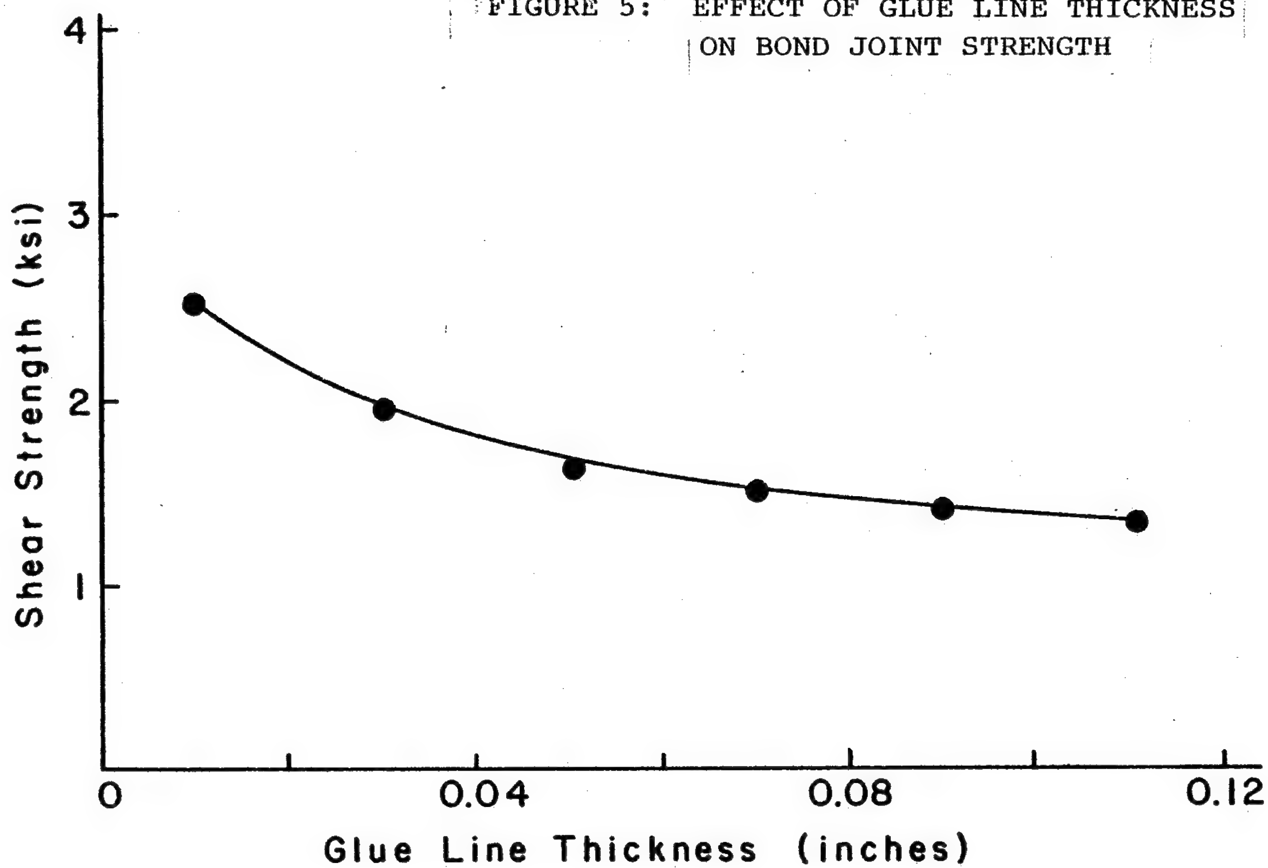
In order to utilize this adhesive system only minimal surface preparation is necessary. The two surfaces to be bonded are prepared with a plastic wash primer (Pliogrip 6033/6034 Wash Primer) that is applied with a cloth. No sand blasting or surface stripping is necessary. To maintain reliably bonded parts, Pliogrip 6000 must be mixed at a precise ratio of 4 parts resin to 1 part curative by weight or volume. Deviations from this standard will yield resin-rich areas of uncured adhesive. The actual mixing of the two components must be carried out without the introduction of air into the system, thus the need for specialized equipment. Without this precaution, entrapped air bubbles in the cured adhesive would yield voids and greatly affect the performance of the bond. Curing this adhesive system can be accomplished at room temperature, however the use of heated fixtures will reduce cure times.

Recommended clamping pressures of heated fixtures range from 20 to 40 psi.

An important criterion in the design of bonded joints is that of the adhesive thickness. It has been shown that adhesive properties vary inversely with adhesive thickness. Thus the bulk properties of the adhesive are distinctly different from those in the film state. So the question is posed as to the optimum bond thickness as a function of shear strength. Figure 5 (taken from Pliogrip technical data, Goodyear Adhesives) shows the effect of glue line thickness on bond joint strength. A bond line thickness of 0.030 inches was chosen as optimal even though thicknesses less than 0.030 inches yield greater bond strengths. It was felt that bonding thicknesses less than 0.030 inches are not capable of being fabricated with consistency under production operations. (i.e. molded FRP parts will inherently not fit together with reliable precision).

To achieve uniform bond lines, one of two procedures is generally used. Adherents may have a small raised button of 0.030 inches in thickness which acts as a spacer for the joint to insure a uniform bond. Another procedure is to introduce small glass spheres (0.030 inches dia.) directly into the adhesive to achieve similar spacing. The effect of these spheres on joint strength has not been determined but it is argued that the variation from the norm is negligible.

FIGURE 5: EFFECT OF GLUE LINE THICKNESS ON BOND JOINT STRENGTH



III. Methods of Analysis

A. Tensile Loading

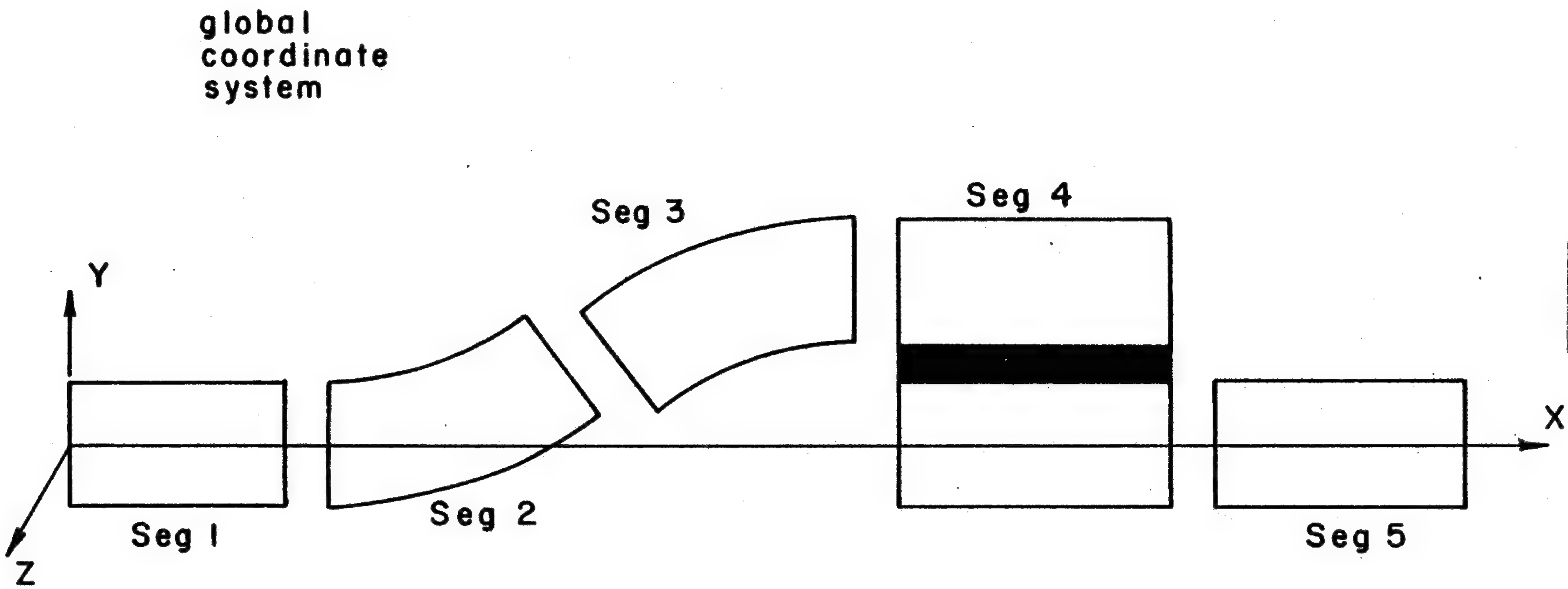
a. Beam Model

Recently, Adkins [ref 2] investigated the response of a scarf joint to simple tensile loadings. It was found that the scarf joint exhibits flexural deformation under tensile loading due to the misalignment between the neutral surface and the loading axis. This eccentricity induces a moment distribution along the joint (see Figure 9) which acts to align the neutral surface with the loading axis.

The analysis of the "joggle-lap" joint, shown previously in Figure 1, is an extension of the concept discussed above. Again it is clear that under tension the joint will experience a lateral deflection as the neutral axis attempts to align with the applied force. To analyze the joint behavior under tensile loading conditions, it was decided to divide the joint into five segments. The obvious places to divide the joint are illustrated in Figure 6 along with the corresponding identifying labels and global coordinate system. Reference to beam segments via their identifying numbers will be utilized throughout the remainder of this analysis.

In general, the goal of the analysis will be to determine the displacements of the neutral axis as measured perpendicularly from the undeformed neutral surface. Once

FIGURE 6: PIECEWISE REPRESENTATION OF THE JOGGLE LAP JOINT



the deflections are known, one may calculate a moment distribution along the joint and thus determine the stress distribution at any given cross-section.

The initial intention of such an investigation was to develop a closed form solution for the stresses within the joint. This effort was soon thwarted by the non-linearities encountered in the governing equations for the beam elements. These non-linearities result from a coupling between the moment and deflection solutions, as will be evident later. As an alternative solution, the displacement field was obtained via numerical integration routines.

Linear elastic beam theory states that for a beam under general loading conditions, the local radius of curvature is given by

$$R = \frac{EI}{M} \tag{3}$$

where
R = radius of curvature
E = modulus of elasticity
I = moment of inertia about the neutral surface
M = applied moment

The radius of curvature may be written in terms of the lateral deflection as given by Eq. (4)

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{[1 + (\frac{dy}{dx})^2]^{3/2}} \tag{4}$$

Realizing that under the assumptions made with regard to small deflection theory, the term $(\frac{dy}{dx})^2$ will be negligible when compared to unity. Thus one arrives at the governing equation for straight beam elements.

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{5}$$

Since the material system is relatively stiff, it is assumed that small deflection beam theory will yield sufficiently accurate results. Thus, one may write a governing differential equation for each segment of the joint. By matching boundary conditions of deflection and slope at each interface, the deflection of the entire joint may be obtained as a function of distance along the neutral axis. Details of the analysis may be referenced in Appendix B.

To enhance one's understanding of the joint behavior under applied tensile loadings, Figures 7 through 10 show deflection, slope, moment, and shear diagrams respectively at a load of 200 lbs. Many of the discontinuities found in the plots arise from a shift in the neutral axis which is a common occurrence among lap joints.

It was stated previously that analyzing the "joggle-lap" joint under tension was a non-linear problem. This was seen by the fact that the moment was a function of the deflection. Another way to view the non-linearities of the

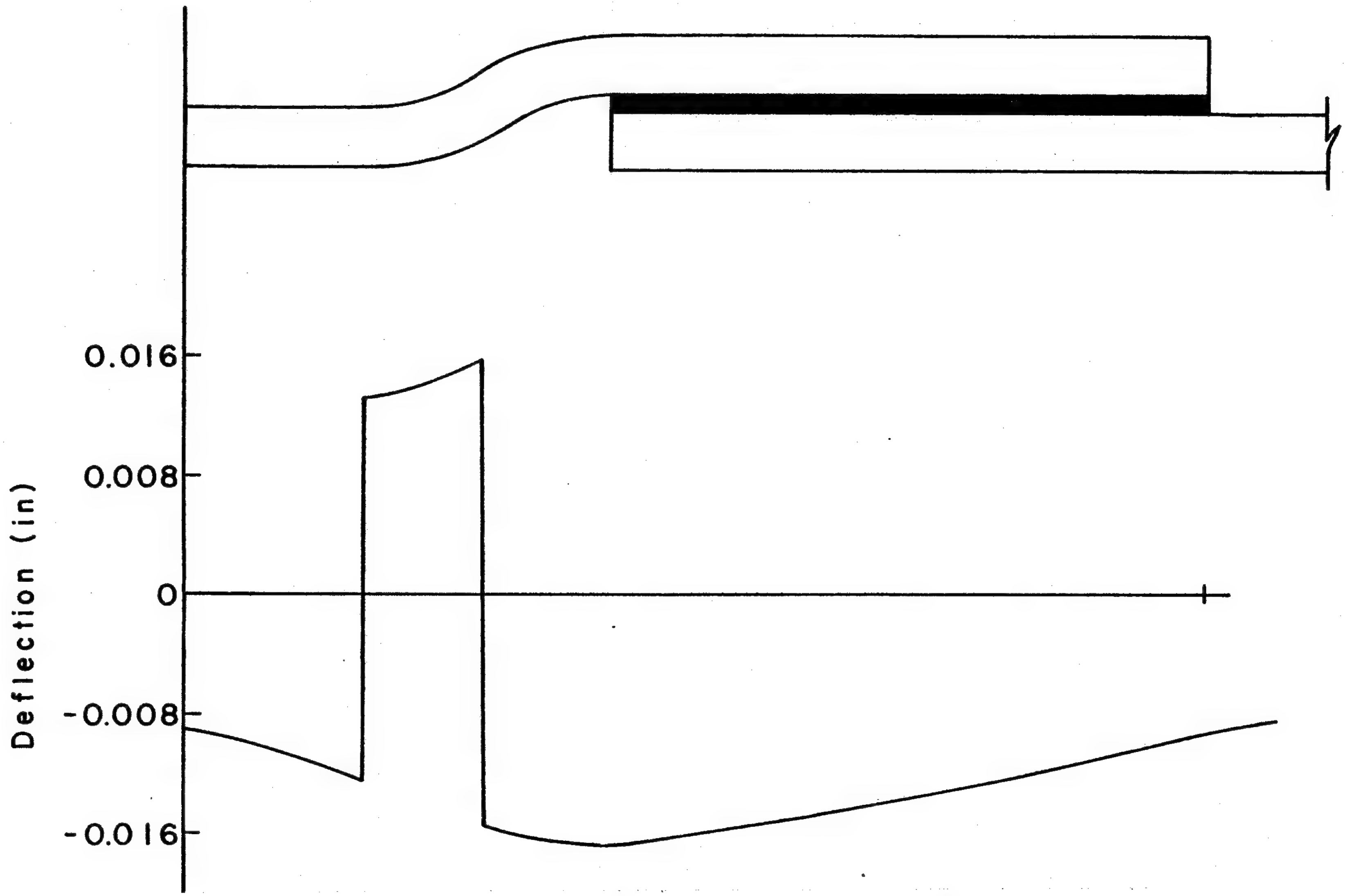


FIGURE 7: DEFLECTION OF NEUTRAL SURFACE AT FAILURE LOAD

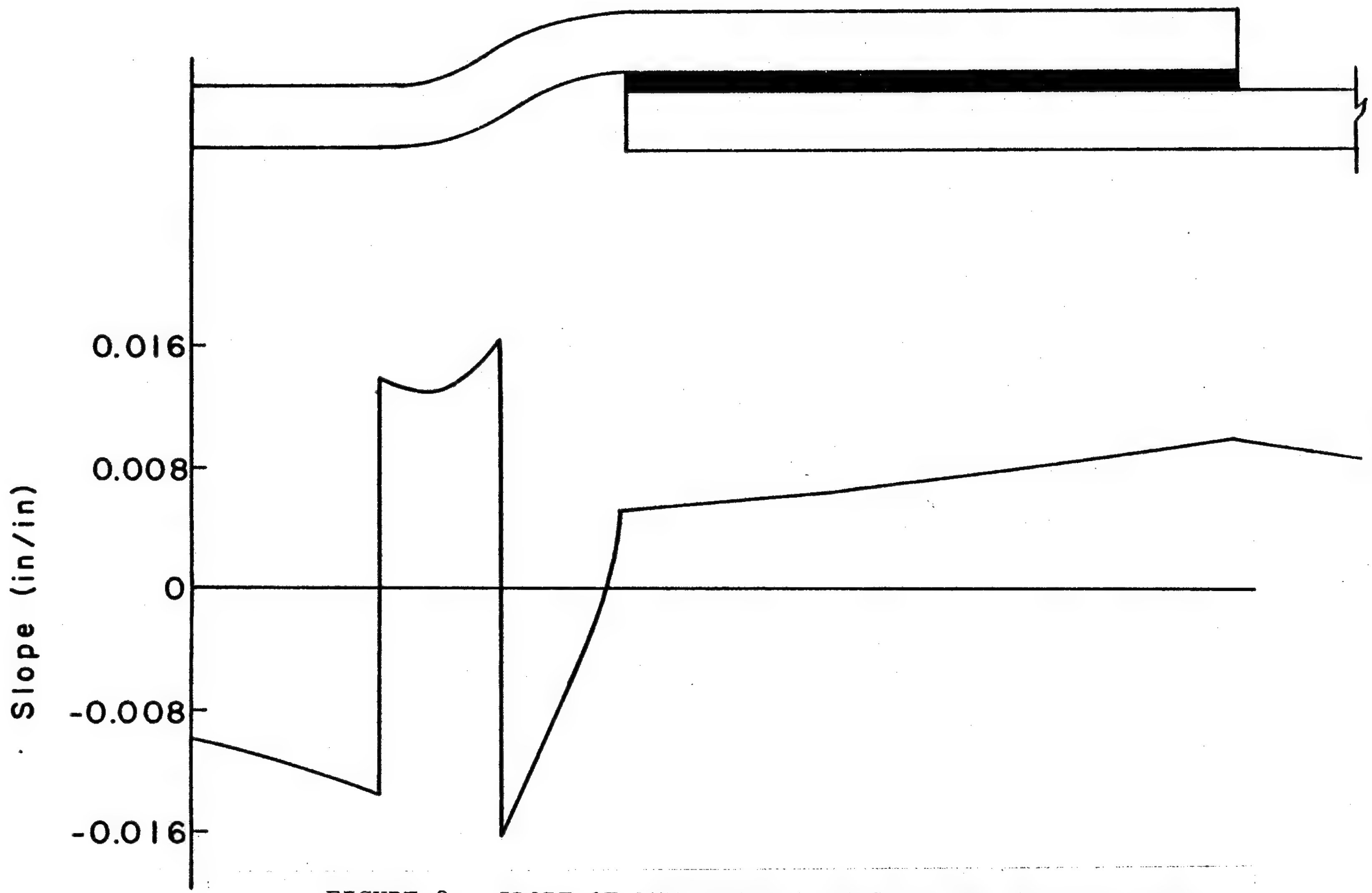


FIGURE 8: SLOPE OF THE NEUTRAL AXIS AT THE FAILURE LOAD

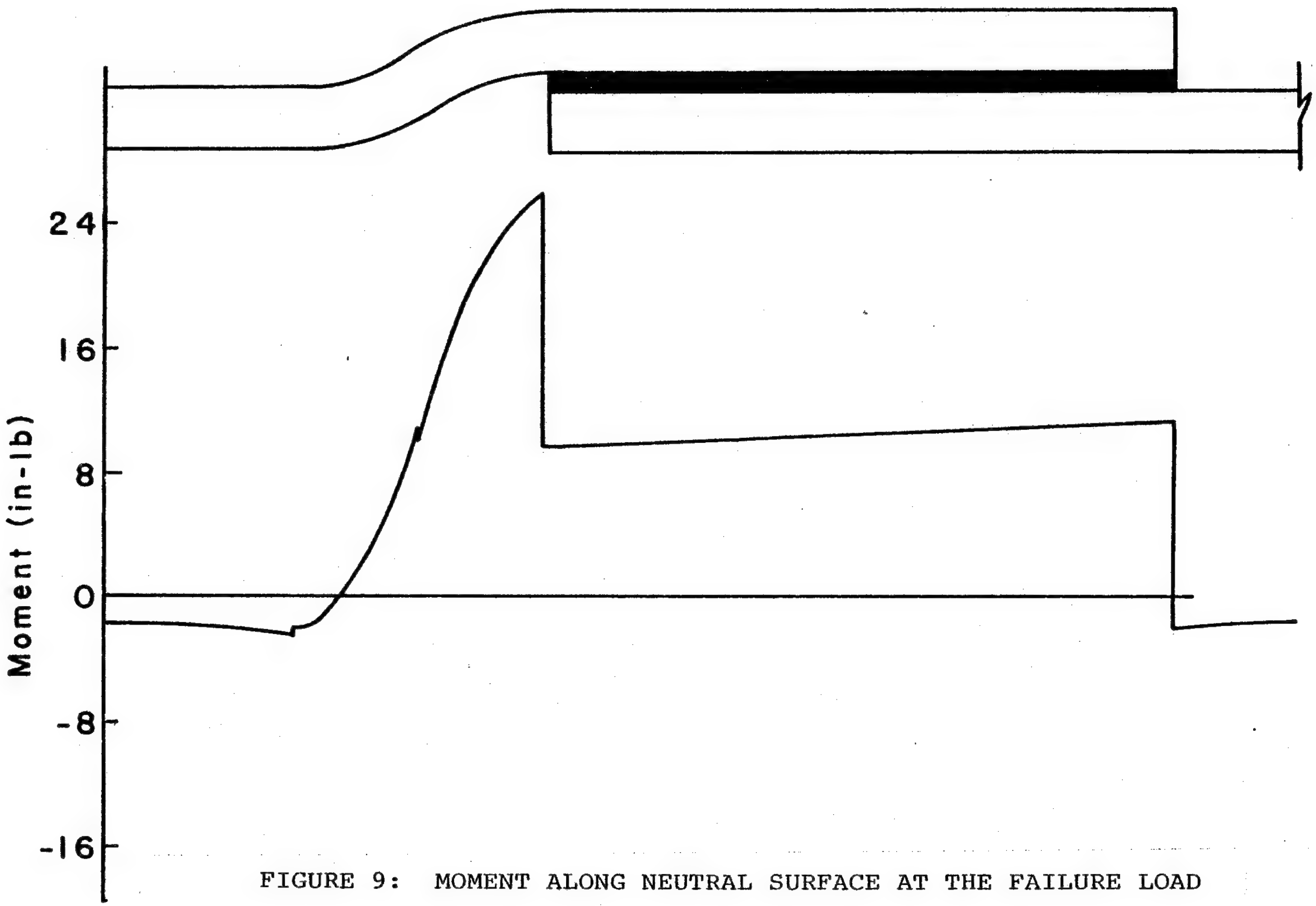


FIGURE 9: MOMENT ALONG NEUTRAL SURFACE AT THE FAILURE LOAD

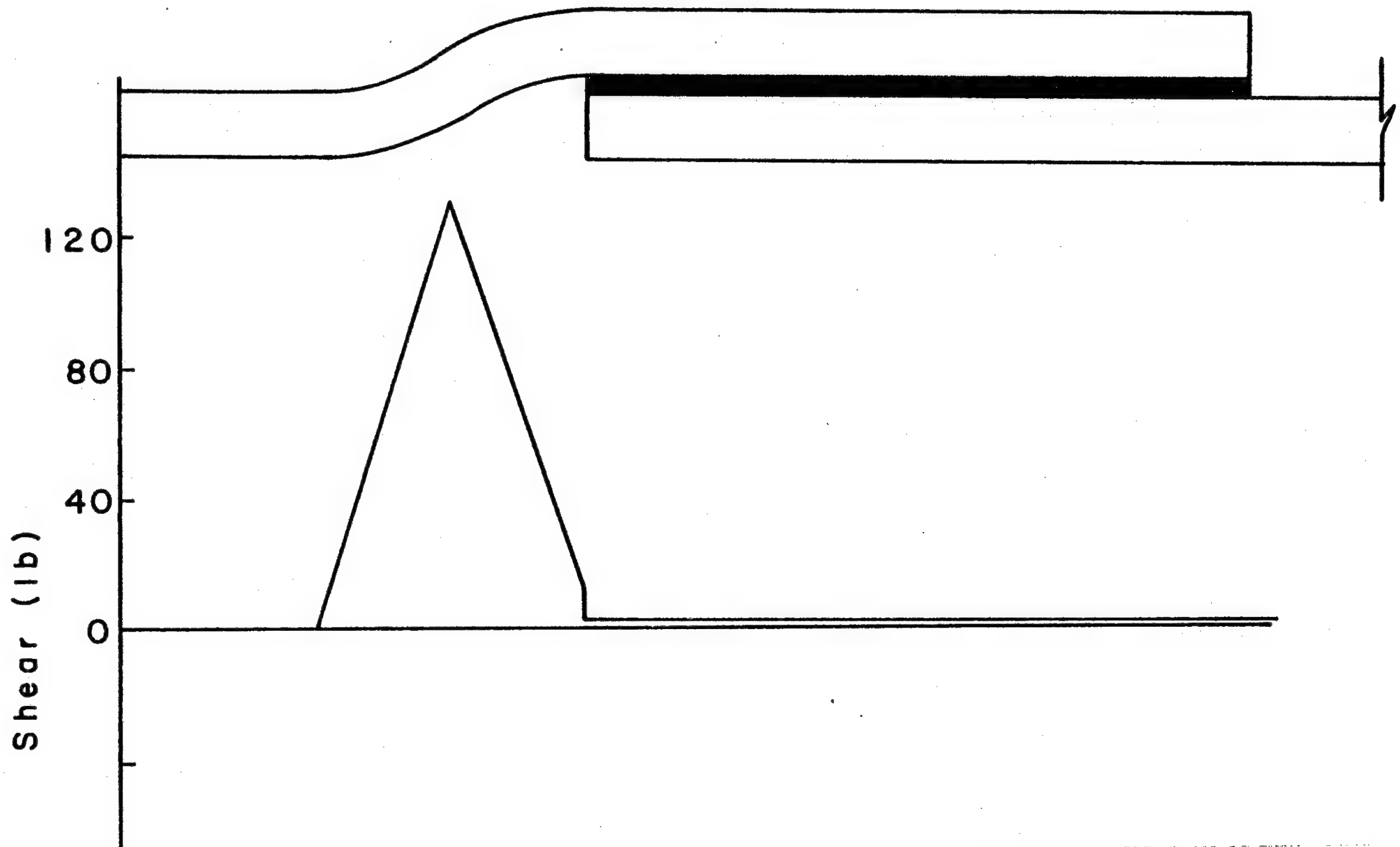


FIGURE 10: SHEAR ALONG THE NEUTRAL SURFACE AT THE FAILURE LOAD

joint behavior is to investigate the response of the joint to varying tensile loads. Figures 11 and 12 provide a clear indication of the deviation from linearity even for small values of load. Both the deflection (Figure 11) and moment (Figure 12) were recorded at the beginning of SEG3. (i.e. $S_3 = 0$)

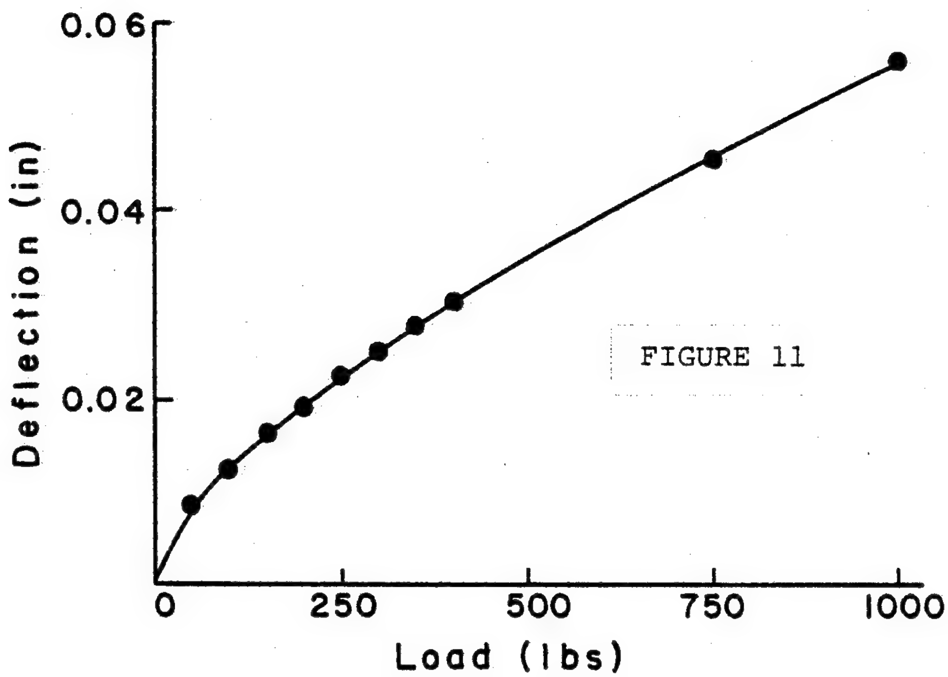


FIGURE 11

RESPONSE OF THE JOGGLE-LAP JOINT AT $S_3 = 0$

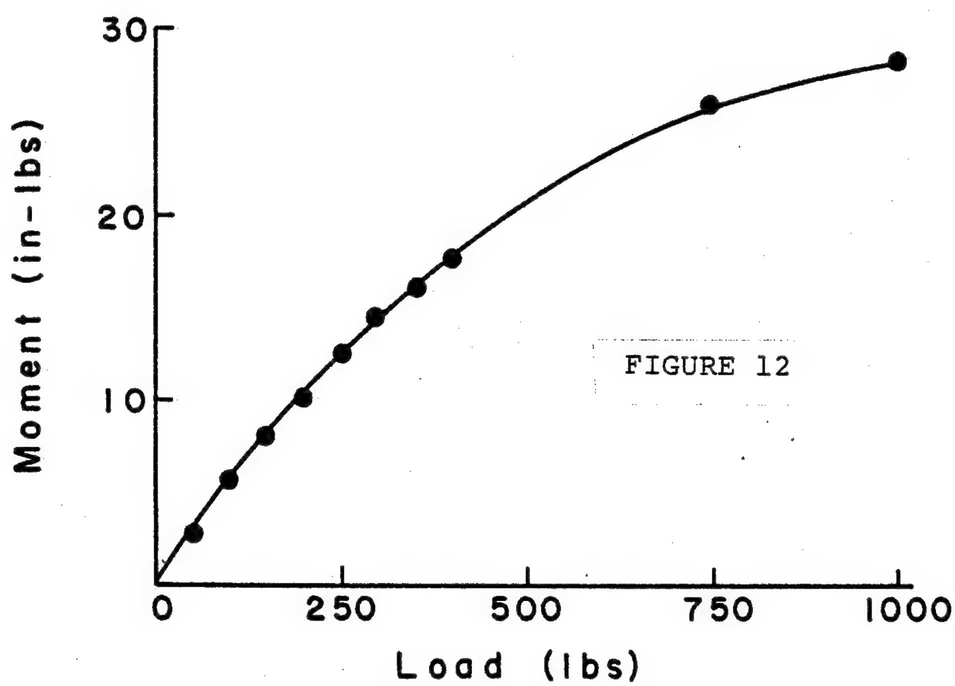


FIGURE 12

b. Finite Element Model (tension)

Anticipating the shortcomings of a beam bending model in the adhesive zone, defined to be the area of actual bonding, it was decided to model this area using finite-element methods. One of the underlying assumptions of small deflection beam theory is that plane sections remain plane during pure bending action. Clearly the validity of this assumption is questionable in the bonded area. Another reason for employing the finite element technique was to uncover any local stress concentrations that may not be revealed in a beam analysis. The finite-element mesh, consisting of 7 material types, is shown in Figure 13. Boundary conditions in the form of concentrated loads were applied to each of the finely meshed ends. Loading conditions were applied away from the adhesive layer at a distance of 1.5 times the thickness in an effort to minimize the effects of the end loads upon the stress solution. An explanation of how these boundary conditions were determined will follow shortly. A plane stress analysis was utilized to calculate the displacement and stress fields. Figures 15 through 17 are the result of a plotting routine which displays lines of constant stress. The figures should be interpreted in the same manner as that of a topographical map. Adjacent lines spaced closely together indicate areas of high stress gradients and possible sites for structural failure. The

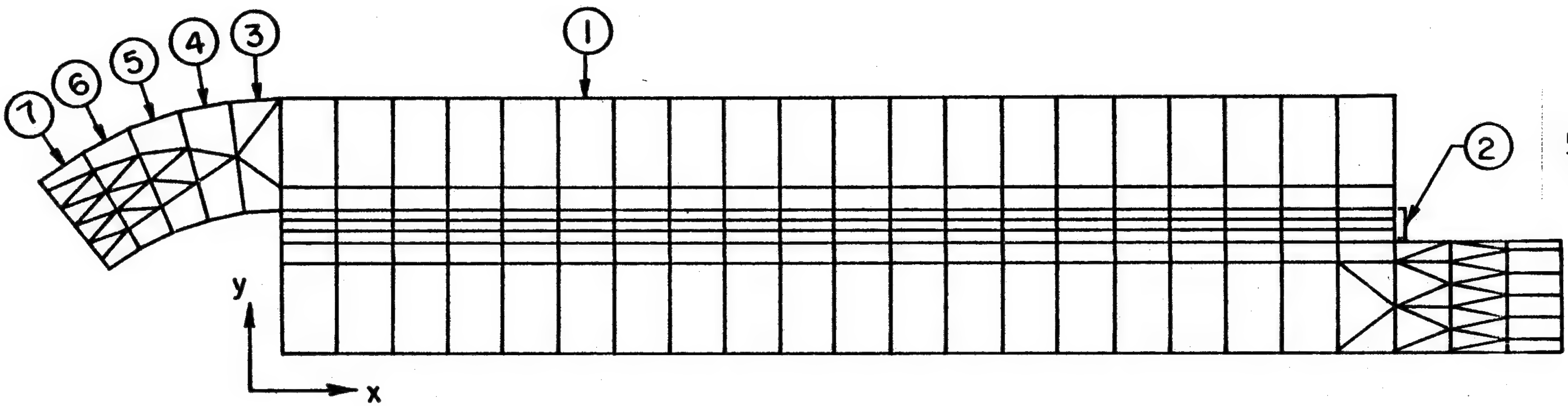


FIGURE 13: FINITE ELEMENT MESH OF THE ADHESIVE ZONE

7 Material Types

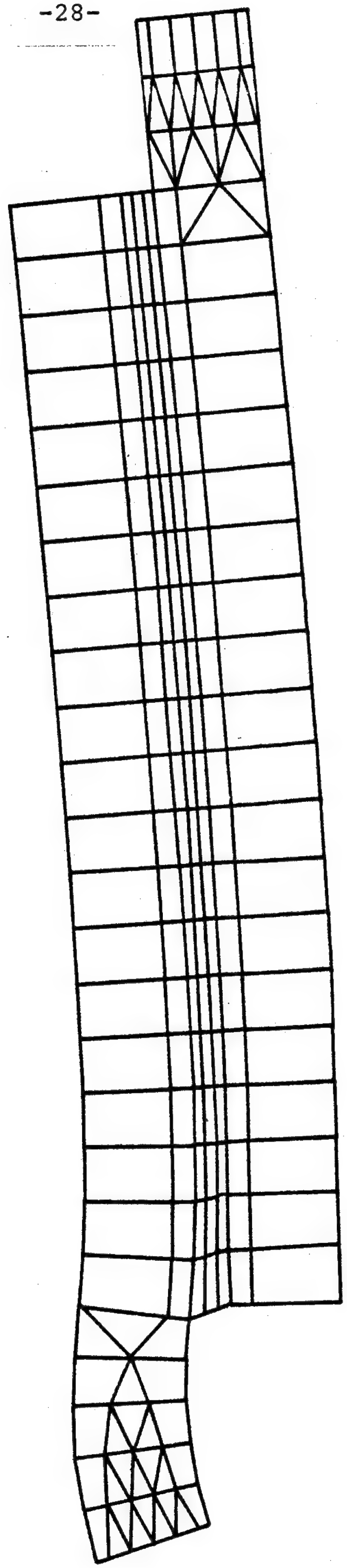


FIGURE 14: DEFORMED MESH AT THE FAILURE LOAD

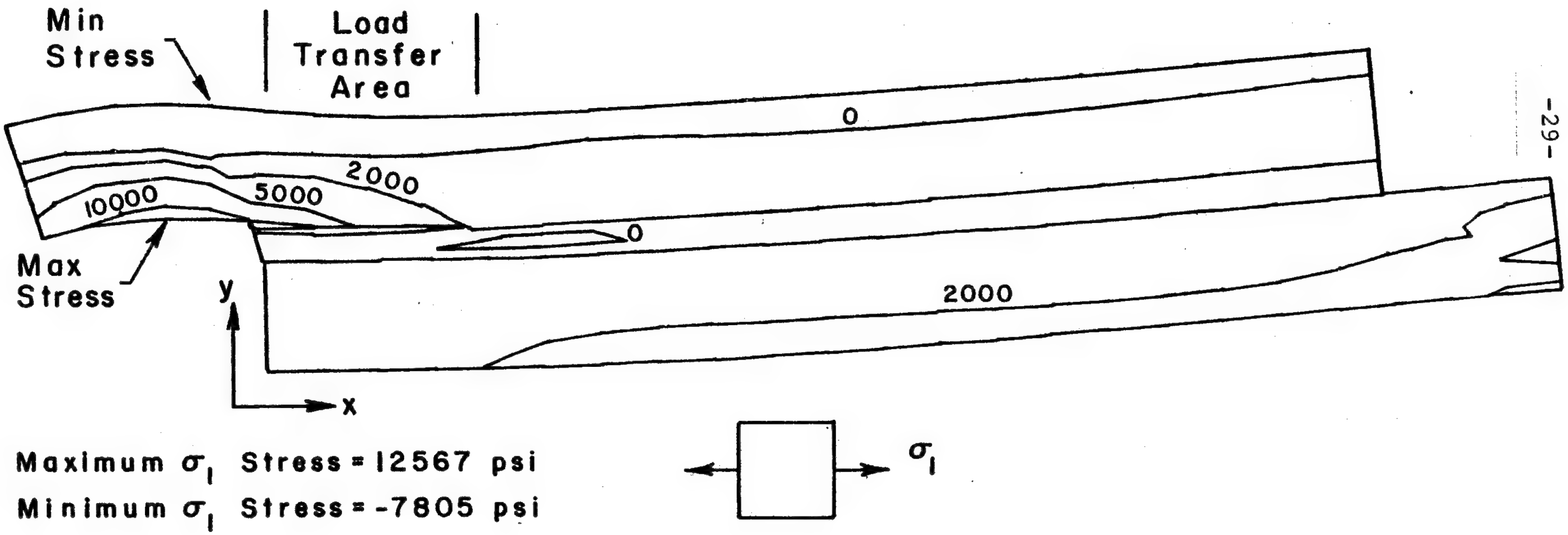


FIGURE 15: THE ADHESIVE ZONE IN TENSION - CONTOURS OF σ_1 STRESS

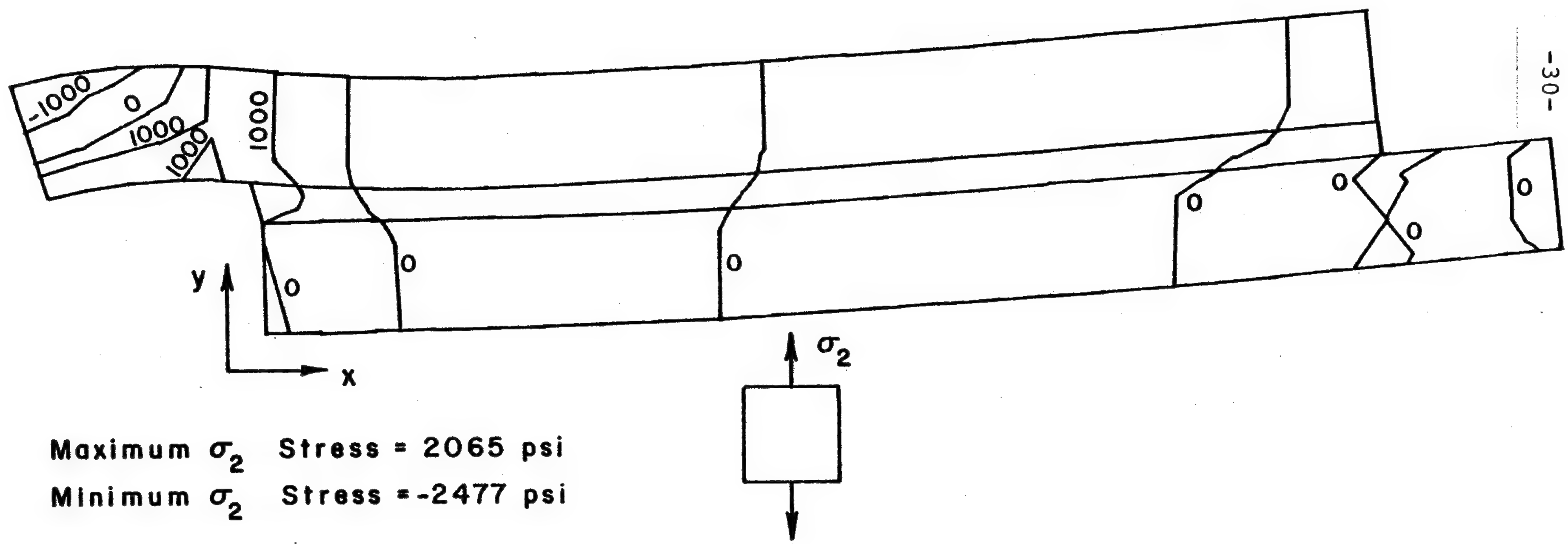


FIGURE 16: THE ADHESIVE ZONE IN TENSION - CONTOURS OF σ_2 STRESS

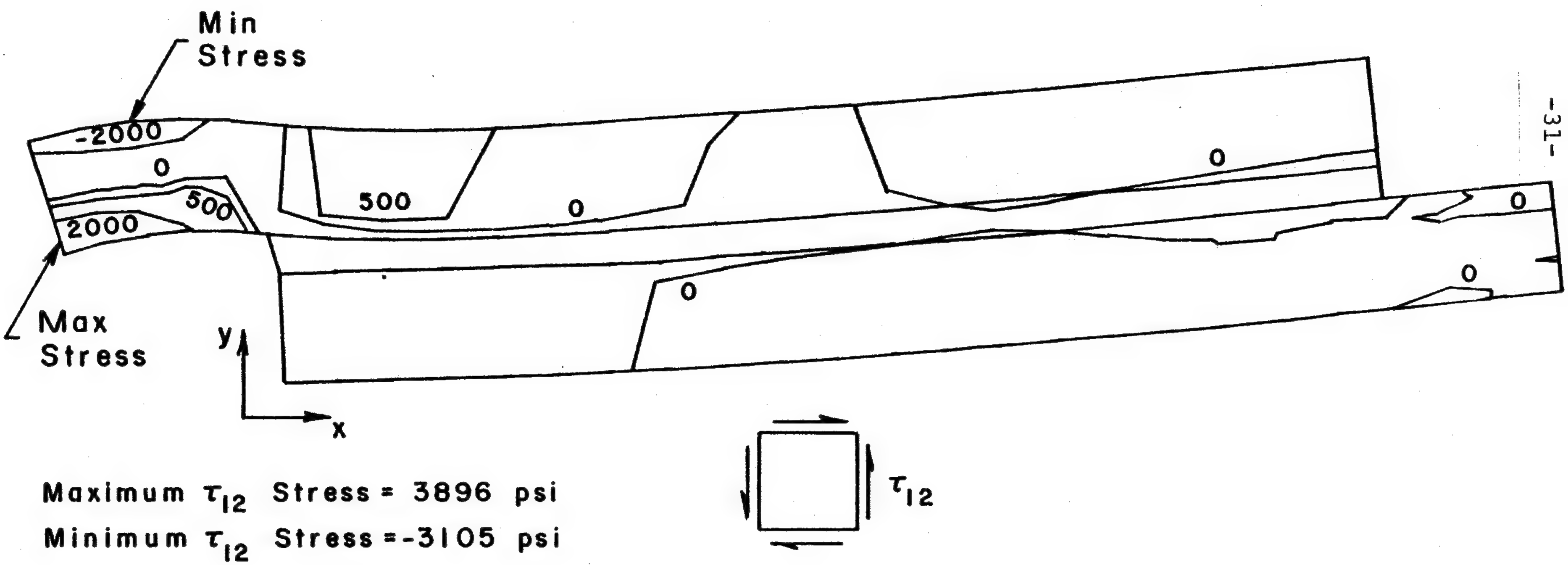


FIGURE 17: THE ADHESIVE ZONE IN TENSION - CONTOURS OF τ_{12} STRESS

figures are labeled according to the component of stress being displayed. All three plots are the result of loading the specimen at the tensile failure load and are representative of the deformed geometry.

The limitations of the beam bending model are clearly displayed in Figure 15 and reveal the justification for the finite-element model. Shown in the figure is a smooth transition of stress across a change in cross-sectional area, (i.e. shift of the neutral axis) as calculated by the finite-element method. Experimental results have shown this to be a correct representation of the stresses. Beam analysis would have shown a sharp discontinuity in the stress profile where such a shift in the neutral axis occurs. Since the moment is nearly constant throughout SEG4 (see Figure 9) beam analysis would calculate σ_1 stress contours parallel to the adhesive layer. The σ_1, σ_2, and τ_{12} stress components are global oriented stresses as opposed to those that can vary according to element orientation. Marked on each figure are those areas where the assumptions made via beam analysis quite appreciably affect the accuracy of a correct solution.

Many analyses of lap joints assume a condition of constant shear stress in the adhesive layer itself. This would indeed be the case if the adherents were infinitely stiff as compared to the adhesive and also if the existence of a load transfer area was prohibited. Shear stress data

from the finite element model is plotted in Figures 18 and 19 and the indication is clear that the shear stress is not a constant in the load transfer area. The case of constant shear stress found toward the center of the adhesive zone, however, reveals the linear nature of the displacement function through the adhesive thickness in this area.

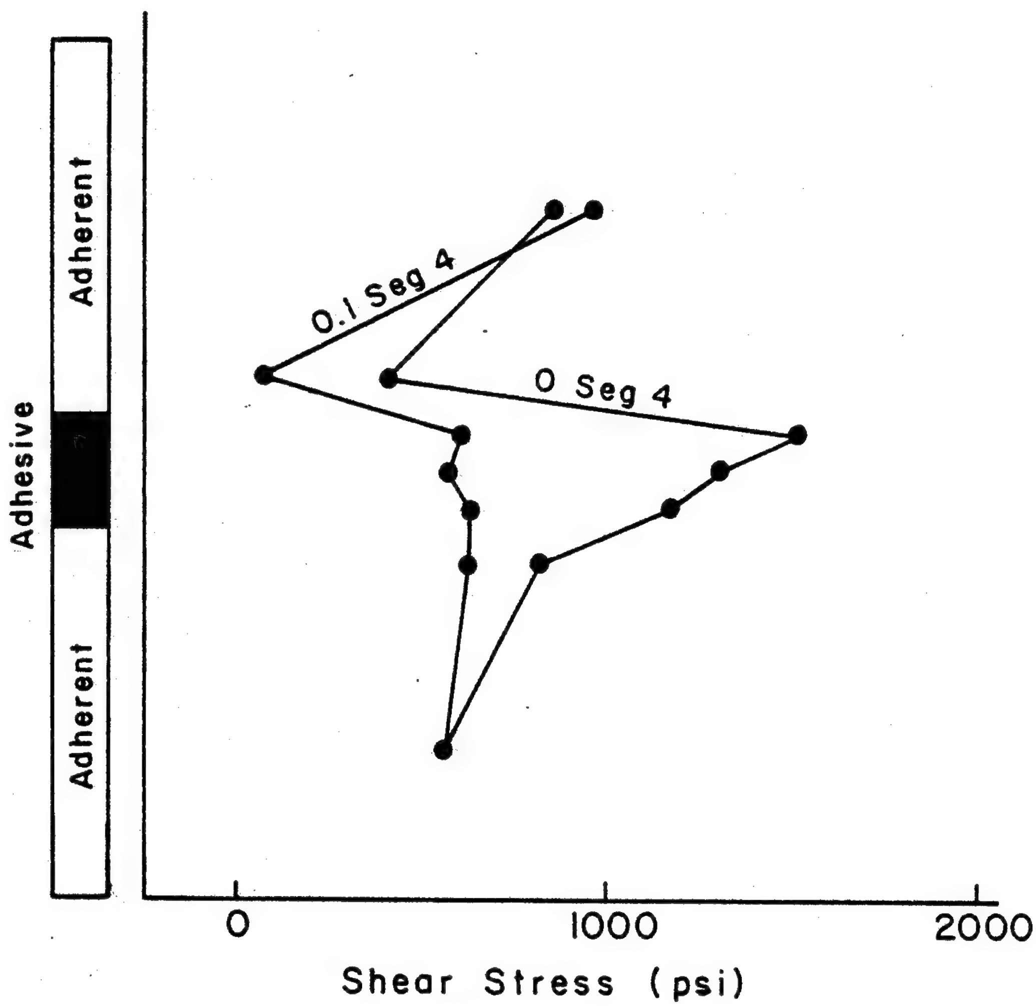


FIGURE 18: SHEAR STRESS VARIATION THROUGH THE THICKNESS OF THE ADHESIVE ZONE

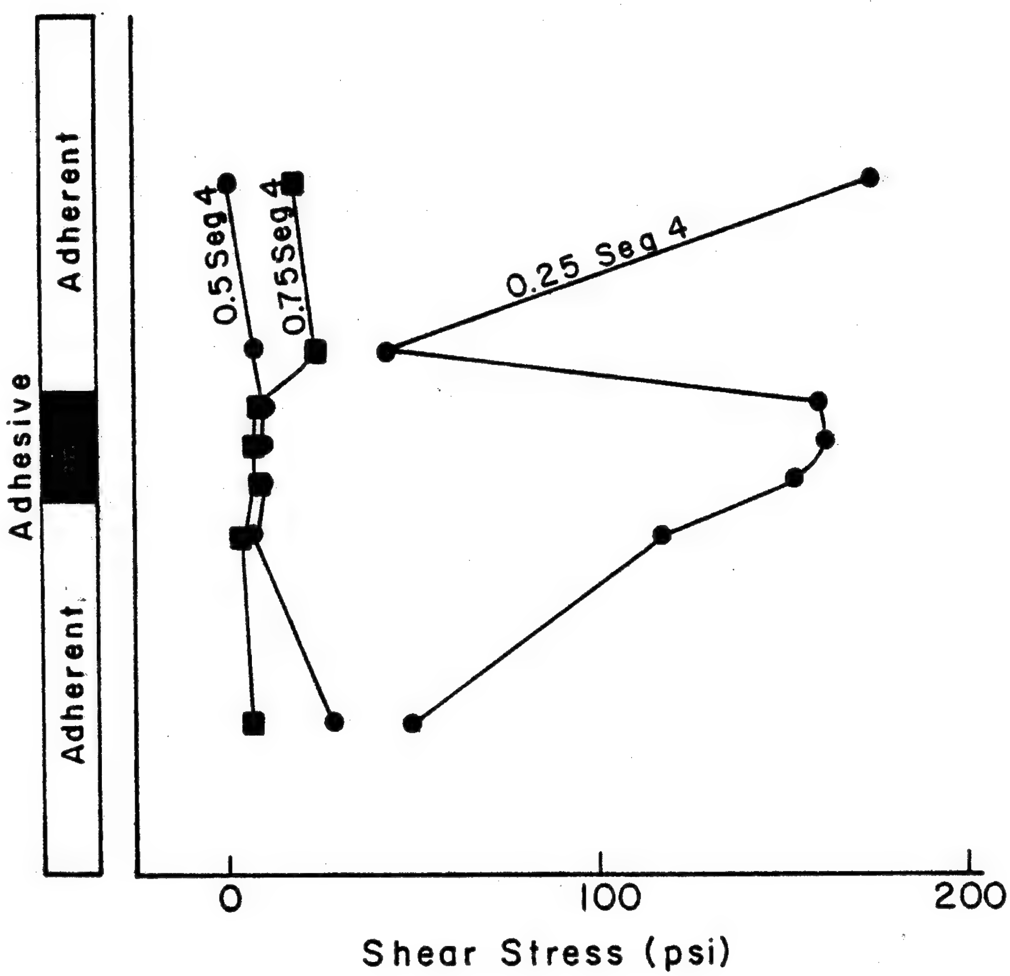


FIGURE 19: SHEAR STRESS VARIATION THROUGH THE THICKNESS OF THE ADHESIVE ZONE

Boundary Conditions for the Finite-Element Model

The boundary conditions for the finite-element model are determined by applying the stress distribution as directed by the beam bending model to the finely meshed ends of the undeformed geometry of the finite element model. In other words, the stresses in the deformed geometry (beam model) must be moved through a distance to their equivalent point of application in the undeformed geometry (finite-element model). The reason for this difficulty with boundary conditions is that we are currently utilizing a linearized finite element routine, SAP V[2], to solve a non-linear problem. Justification of such a procedure will hopefully become lucid with time.

To facilitate the derivation of a transformation routine, Figures 20 and 21 illustrate the following sign conventions. Figure 20 depicts a stress distribution for the left hand face of the finite element model with tension being taken as positive and compression being negative. Note that the neutral axis is not coincident with the centroidal axis inherent in the analysis of a curved beam. As mentioned previously, this fact yields a hyperbolic stress distribution which slightly complicates the computations. (SEE derivations of governing equation for stresses in a curved beam, Appendix A)

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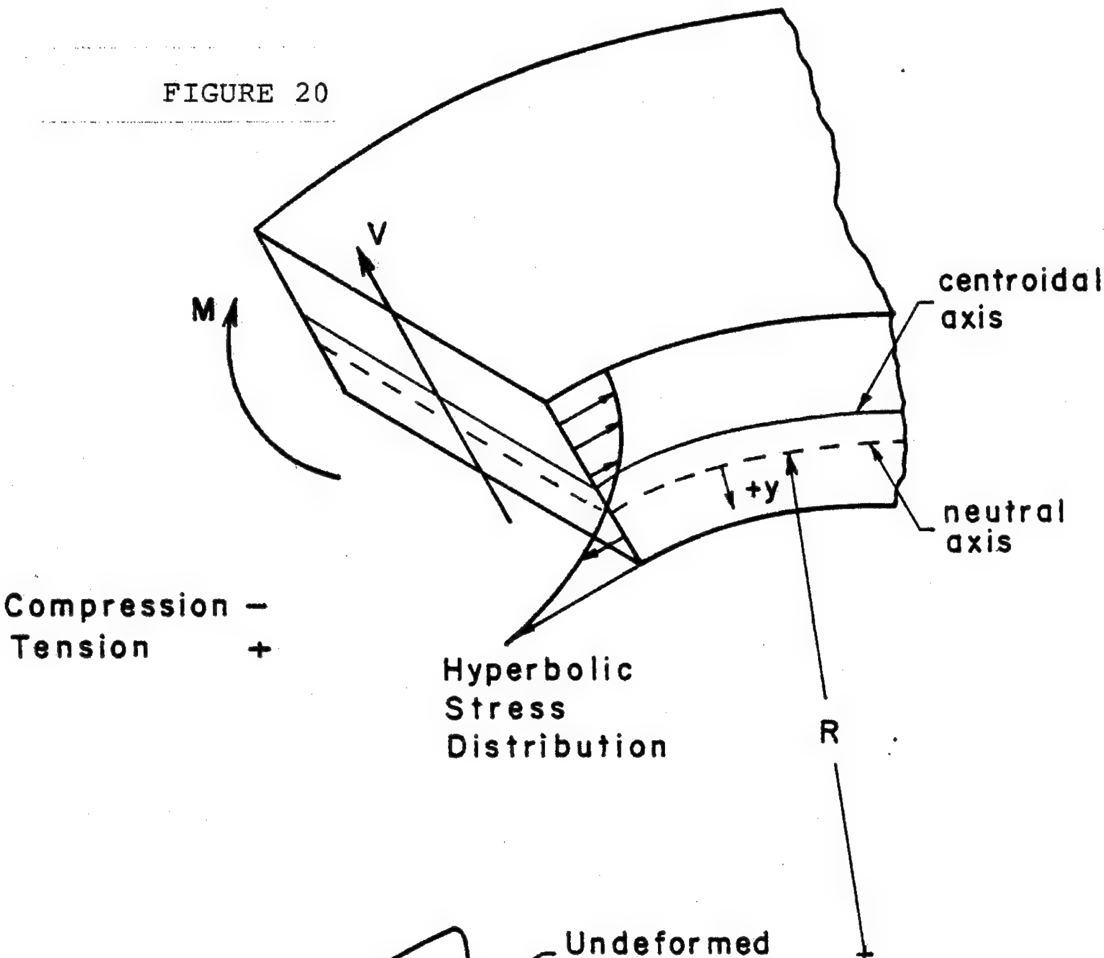
[2]Structural Analysis Program V; University of Southern California, Department of Civil Engineering, Oct. 77.

FIGURE 20



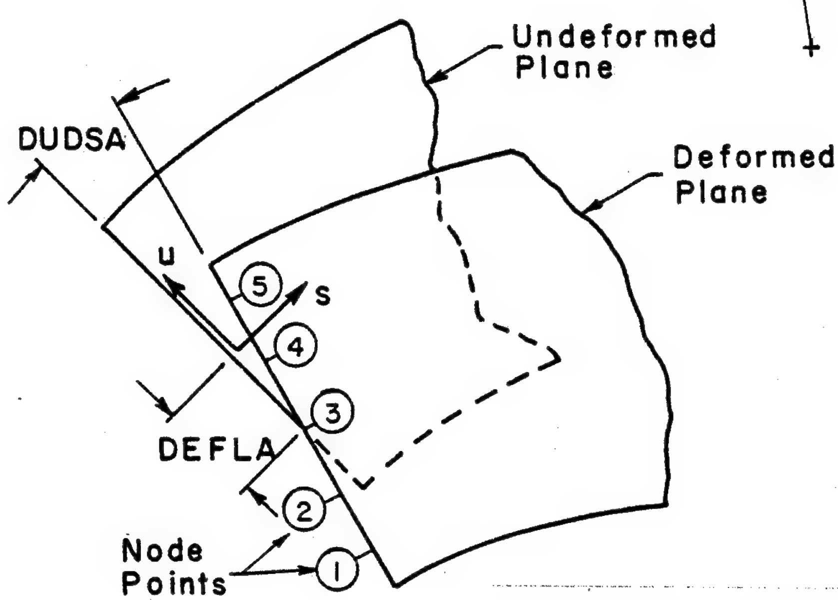


FIGURE 21: SIGN CONVENTION

Figure 21 reveals a planar view of the deformed and undeformed sections. It is assumed in this derivation that the section of the beam can at most undergo a translation and a rotation. Translations are measured via the parameter DEFLA and are positive radially outward as shown. Small deflection theory also allows the rotations to be written as a change in slope. This parameter is DUDSA and is positive counter-clockwise.

With these sign conventions clearly in mind the stress distribution of the deformed geometry may now be resolved into concentrated force components. Representing the hyperbolic stress distribution as equivalent point forces and point couples acting at nodal points labeled 1 through 5 on Figure 21 corresponds mathematically to an integration of the stress distribution between fixed limits.

$$F_{ni} = \frac{M}{\bar{u}\, a} \int_{h_{i-1}}^{h_i} \frac{u}{R-u}\, du + \int_{h_{i-1}}^{h_i} F \cos(\theta + \mathrm{DUDSA})\, du \qquad (6)$$

where $i = 1\text{-}5$

F_{ni} = nodal force component

M = moment

$\bar{u}$ = distance between neutral and centroidal axes

a = cross-sectional area

R = radius of curvature

F = load

θ = angle subtended by SEG3

DUDSA = local slope of deformed neutral axis

The first term of Eq. (6) represents the contribution from the hyperbolic stress distribution. The second term acts to superimpose the component of force due to longitudinal loading.

A correcting moment is calculated for each node to equilibrate the two representations of stress on the section.

$$M_{corr} = \int_{h_{i-1}}^{h_i} \sigma(u)u \, du - F_{ni}u \tag{7}$$

The need for the correcting moment is due to the fact that a distributed force is now represented by a point force as shown in Figure 22.

The next step follows from a translation of the point forces. Elementary statics dictates that a point force may be equivalently represented by the same point force and an added moment to account for the translation from the original line of action.

After carrying out a similar procedure for the stresses at the right hand side of the finite element model, the entire system is set in equilibrium by accounting for the shear acting on each face of the model. The values of shear are obtained directly from the beam bending model. Thus a correct set of boundary conditions has been determined for the finite-element model of the adhesive zone. A computer routine designated by CONVERT was written to calculate appropriate boundary conditions and may be found

in Appendix C.

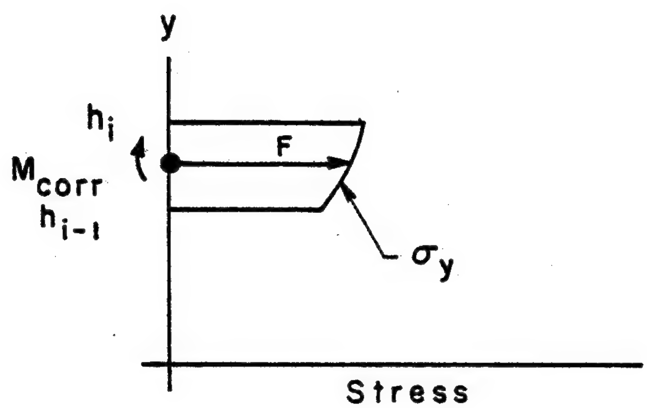


FIGURE 22: ILLUSTRATION OF THE CORRECTIVE MOMENT

Methods of Analysis

B. Flexure Loading

a. Beam Model

The bending behavior of the "joggle-lap" joint was also studied. It was found that the theoretical analysis was far simpler than that encountered for tensile loading. Each segment of the joint (see Figure 6) was modeled as if it were in pure bending. Stresses in the straight beam numbers were calculated via the flexure formula while for the curved beams the formula

$$\sigma_y = \frac{My}{(R-y)\bar{y}a} \tag{8}$$

where M = moment

y = coordinate from the neutral surface (positive radially inward)

R = radius of curvature

$\bar{y}$ = distance between centroidal and neutral axes

a = crossectional area

was used.

In order to compute bending stresses in SEG4 (layered beam) it is necessary to introduce the notion of equivalent sections. In this method we assume all materials to have the same modulus of elasticity. By replacing the actual section with a mechanically equivalent one allows

the flexure formula to be used as a means of computing stresses. The width of the sections are varied so that the new width equals the ratio of the old modulus of the material to the new modulus of the material times the old width as shown in Figure 23. Computing $I_{eq.}$ for the specimen geometry,

$$Ieq = \sum_{i=1}^{3} \left(\frac{1}{12} b_i h_i^{\,3} + a_i d_i^{\,2}\right)$$

b = length of base

h = length of side

a = area

d = distance between element neutral axis and overall section neutral axis

it is apparent that the effect of the adhesive layer on overall section stiffness is negligible. Using the flexure formula and the relation

$$(\sigma_x)_{actual} = \frac{E_{old}}{E_{new}} (\sigma_x)_{equiv.}$$

the stresses in SEG4 may easily be calculated.

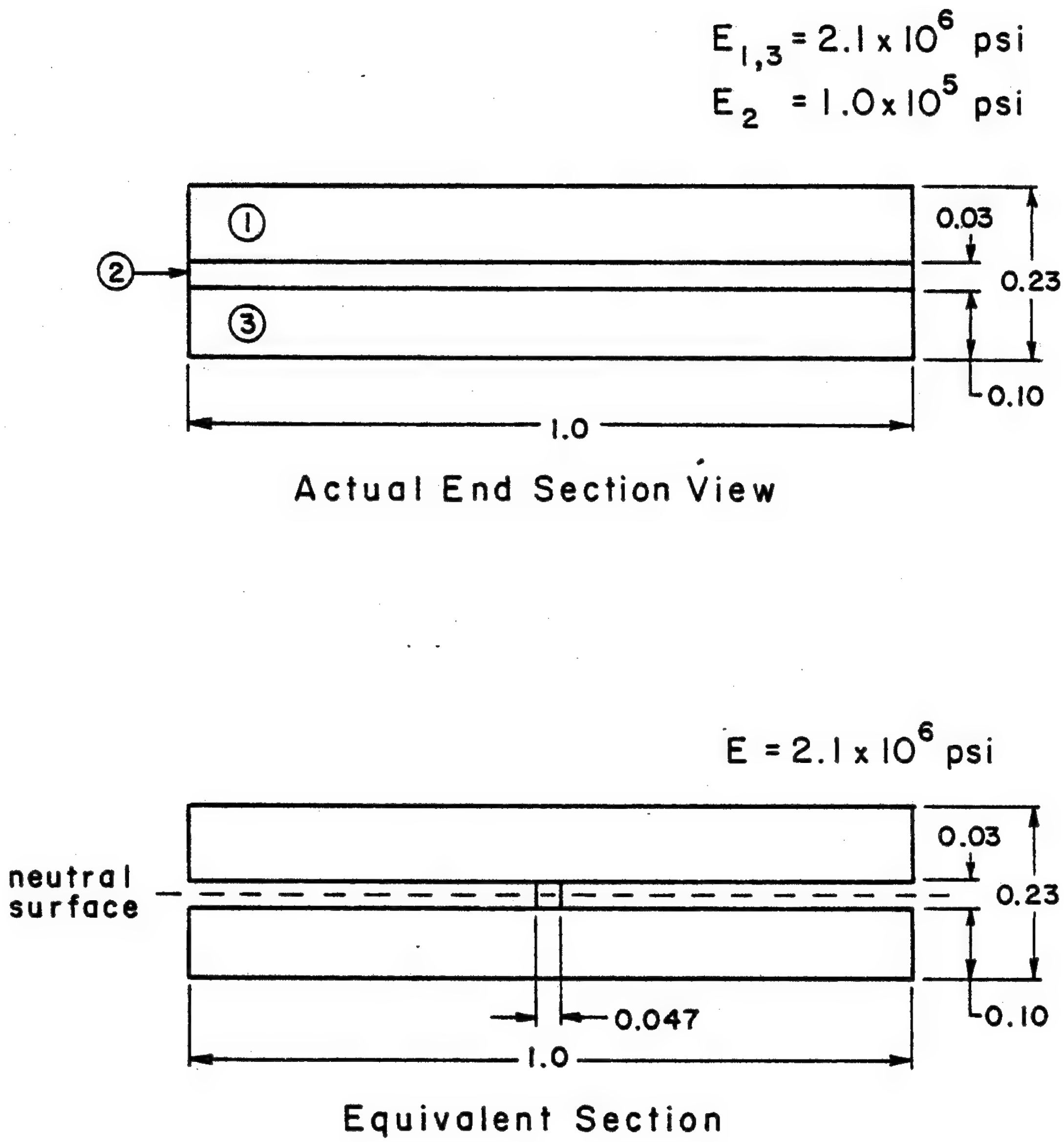


FIGURE 23: METHOD OF EQUIVALENT SECTIONS

b. Finite Element Model (flexure)

The boundary conditions of the finite-element model may be changed to accommodate pure bending. By utilizing couples at the finely meshed ends of the model, stresses in the adhesive zone may be monitored where it has been shown that the results from beam theory are less accurate. Figures 24 through 26 display σ_1,σ_2, and τ_{12} stress contours respectively within the "joggle-lap" joint in pure bending.

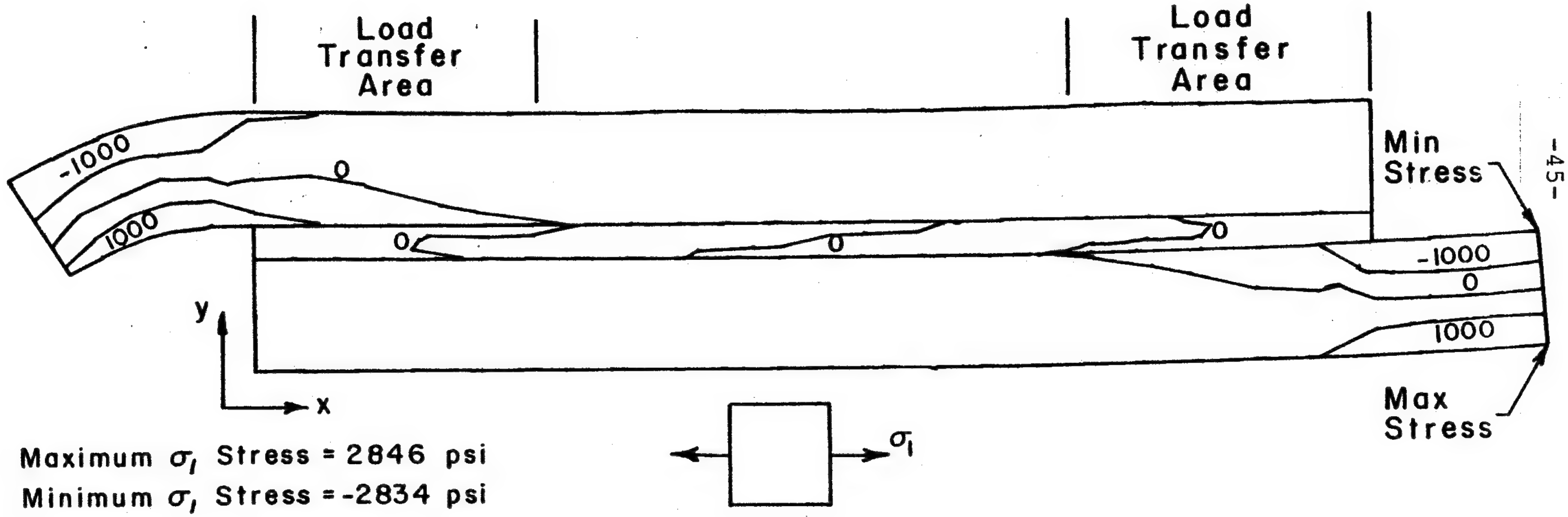


FIGURE 24: THE ADHESIVE ZONE IN BENDING - CONTOURS OF σ_1 STRESS

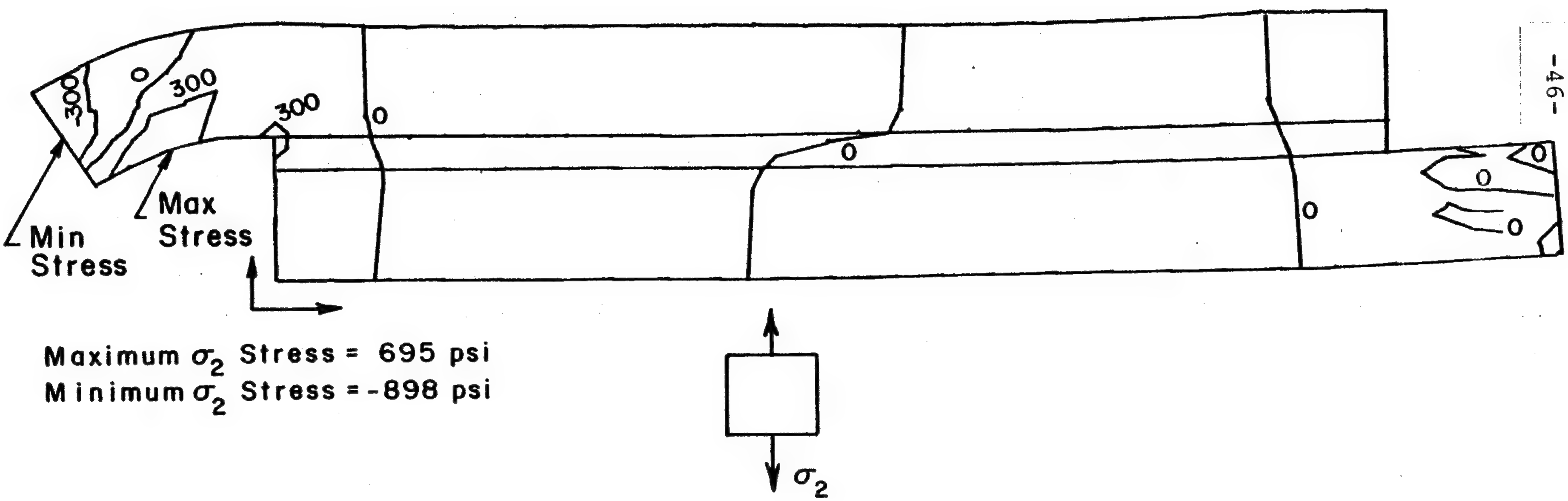
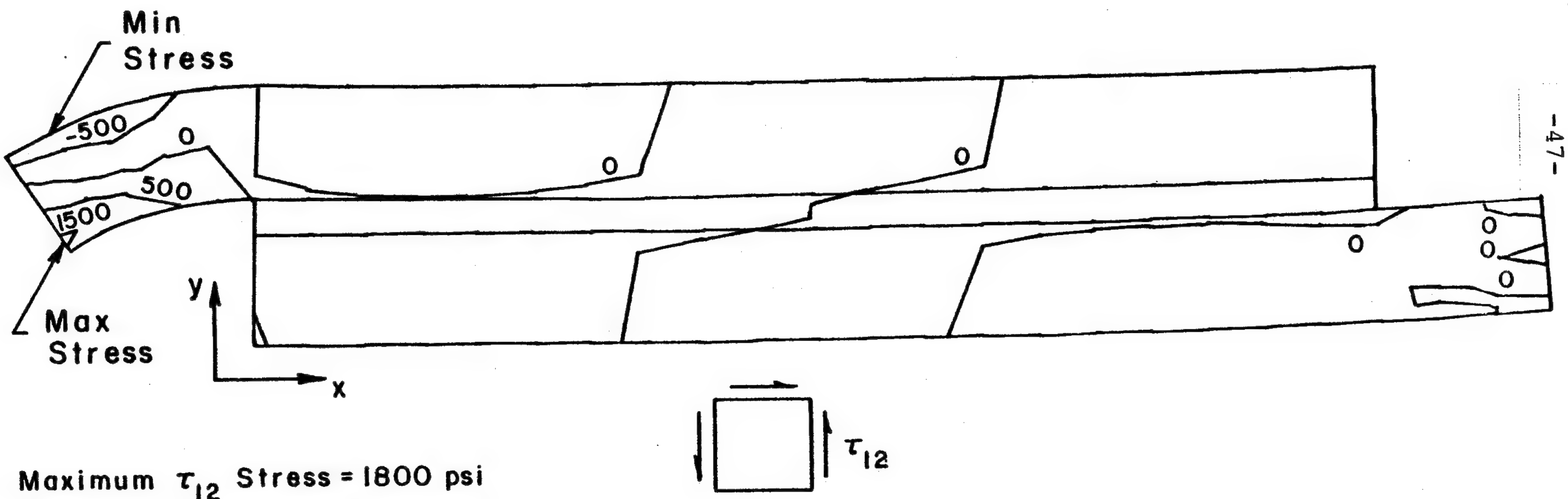


FIGURE 25: THE ADHESIVE ZONE IN BENDING - CONTOURS OF σ_2 STRESS



Maximum τ_{12} Stress = 1800 psi
Minimum τ_{12} Stress = -891 psi

FIGURE 26: THE ADHESIVE ZONE IN BENDING - CONTOURS OF τ_{12} STRESS

IV. Experimental Results

A. Tension

As set forth in the objectives of such a study, an emphasis was to be placed upon developing joint geometries which will accommodate high rate fabrication techniques. In an effort to meet this criterion experimentally, it was necessary to utilize a joint configuration currently being molded in industry. The time and expense of developing in-house molding capabilities proved to be beyond the scope of the research at hand. Thus, test sections were cut from premolded panels of SMC which were later bonded together to form the joint.

The bonding operation was also directed toward high fabrication procedures. All test specimens were adhesively joined at Goodyear Adhesives Division, Ashland, Ohio, via production adhesives application techniques. It was felt that by using these sophisticated application procedures optimum adhesive properties could be obtained.

In general, SMC is defined to be an anisotropic material because of the substantial difference between in-plane and out-of-plane properties. Referring to the coordinate system of Figure 1, the constitutive relations

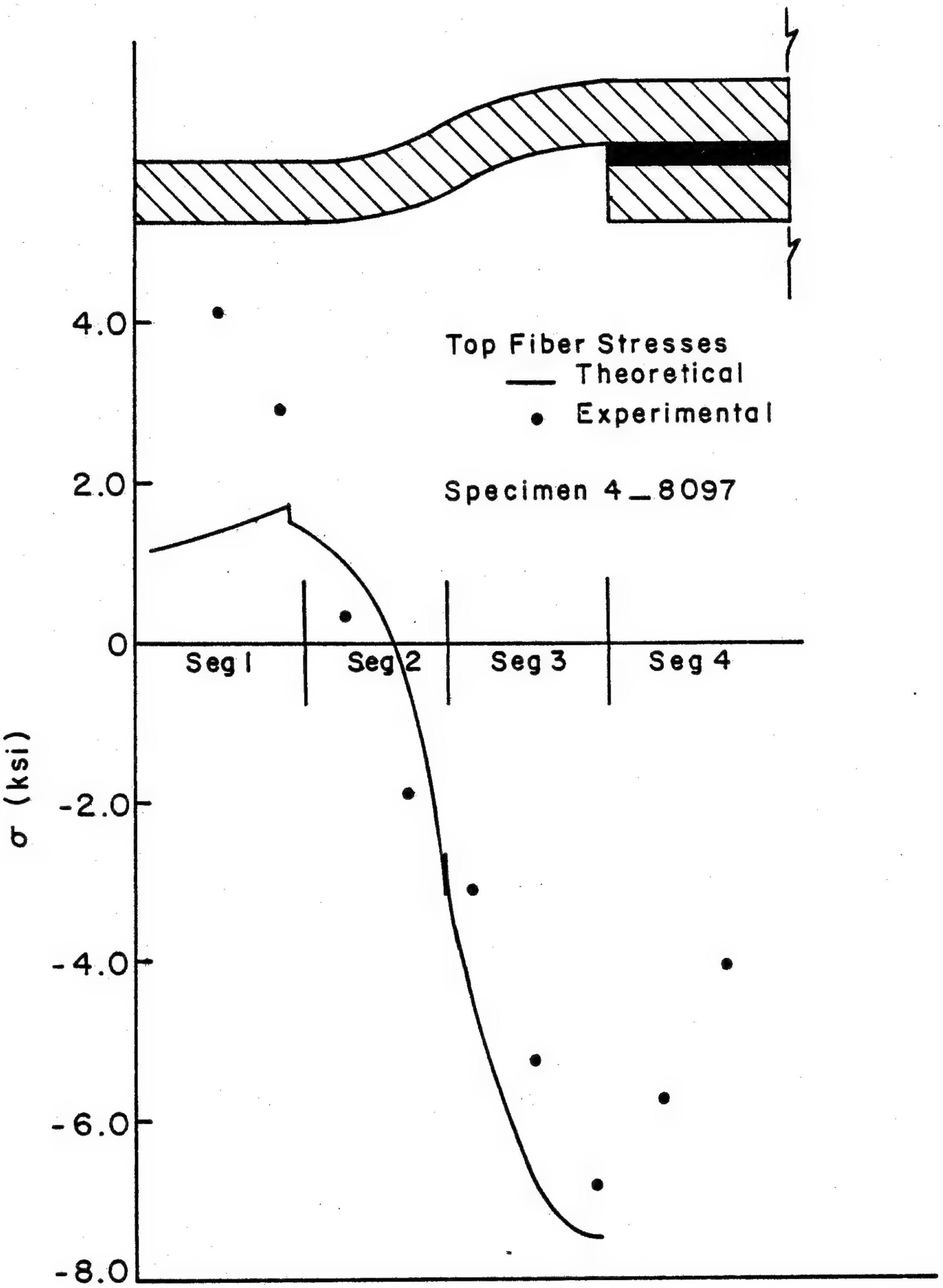


FIGURE 27: ELASTIC RESPONSE DUE TO TENSION - TOP FIBER STRESSES

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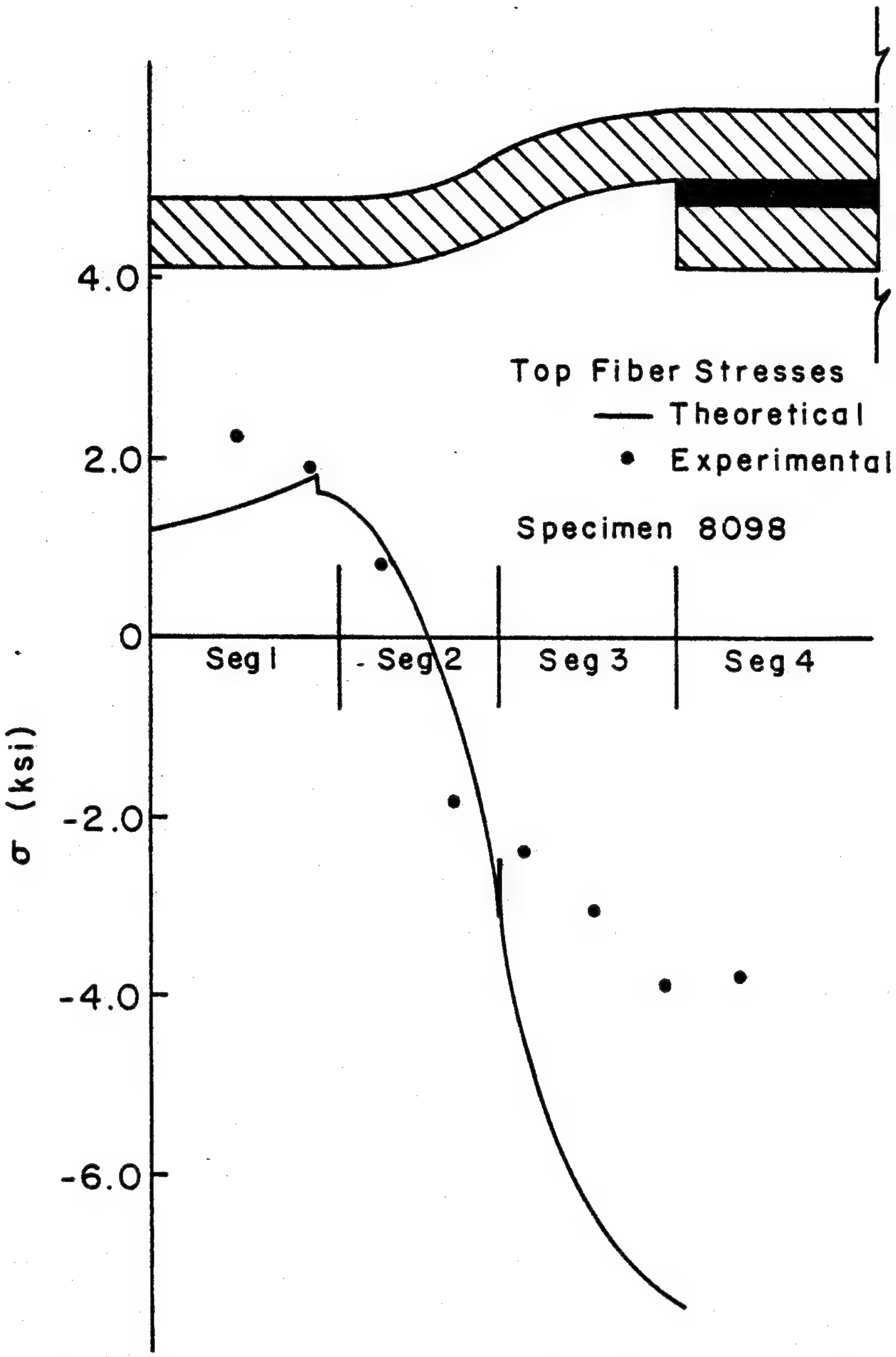

FIGURE 28: ELASTIC RESPONSE DUE TO TENSION - TOP FIBER STRESSES

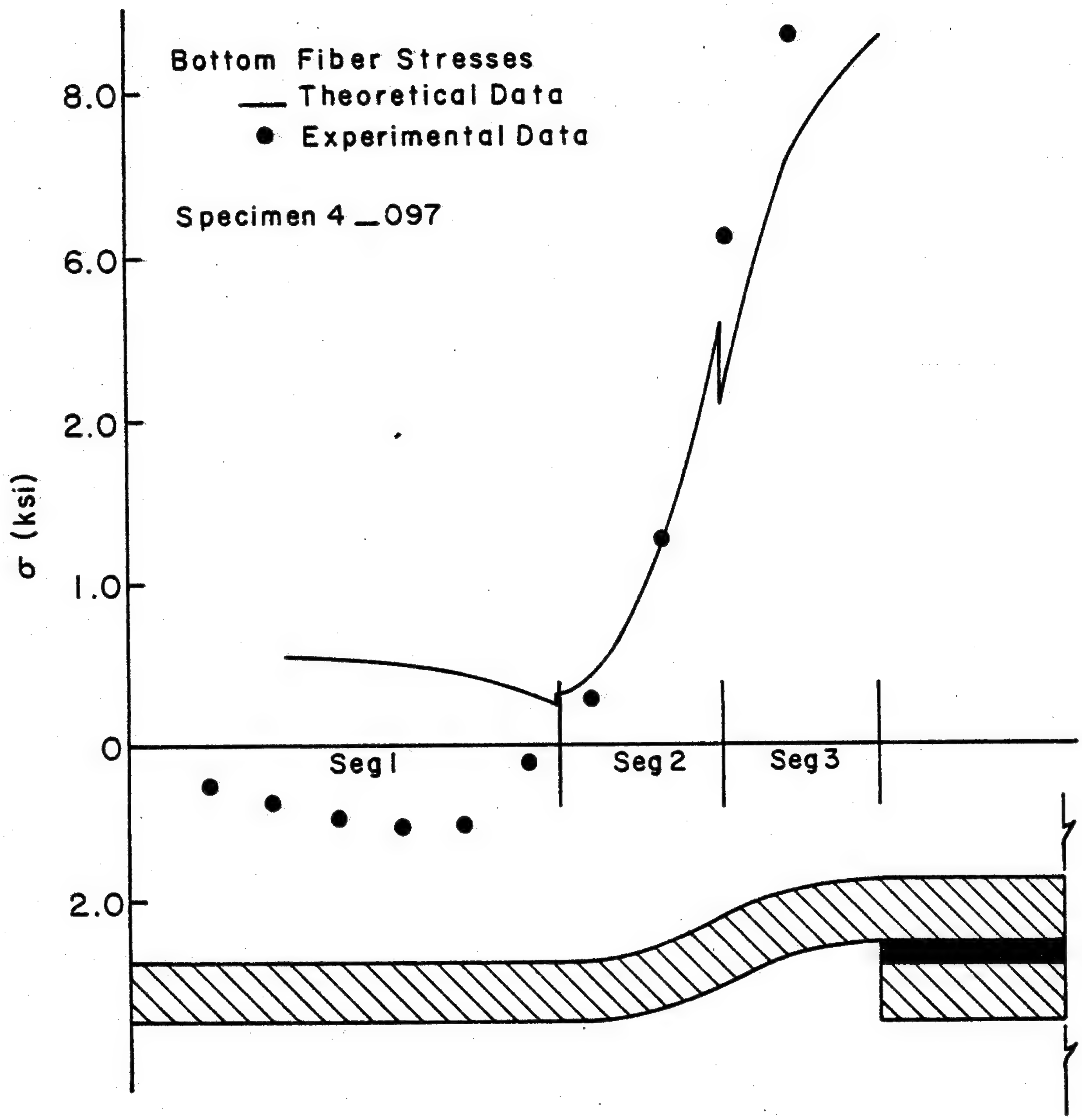

FIGURE 29: ELASTIC RESPONSE DUE TO TENSION - BOTTOM FIBER STRESSES

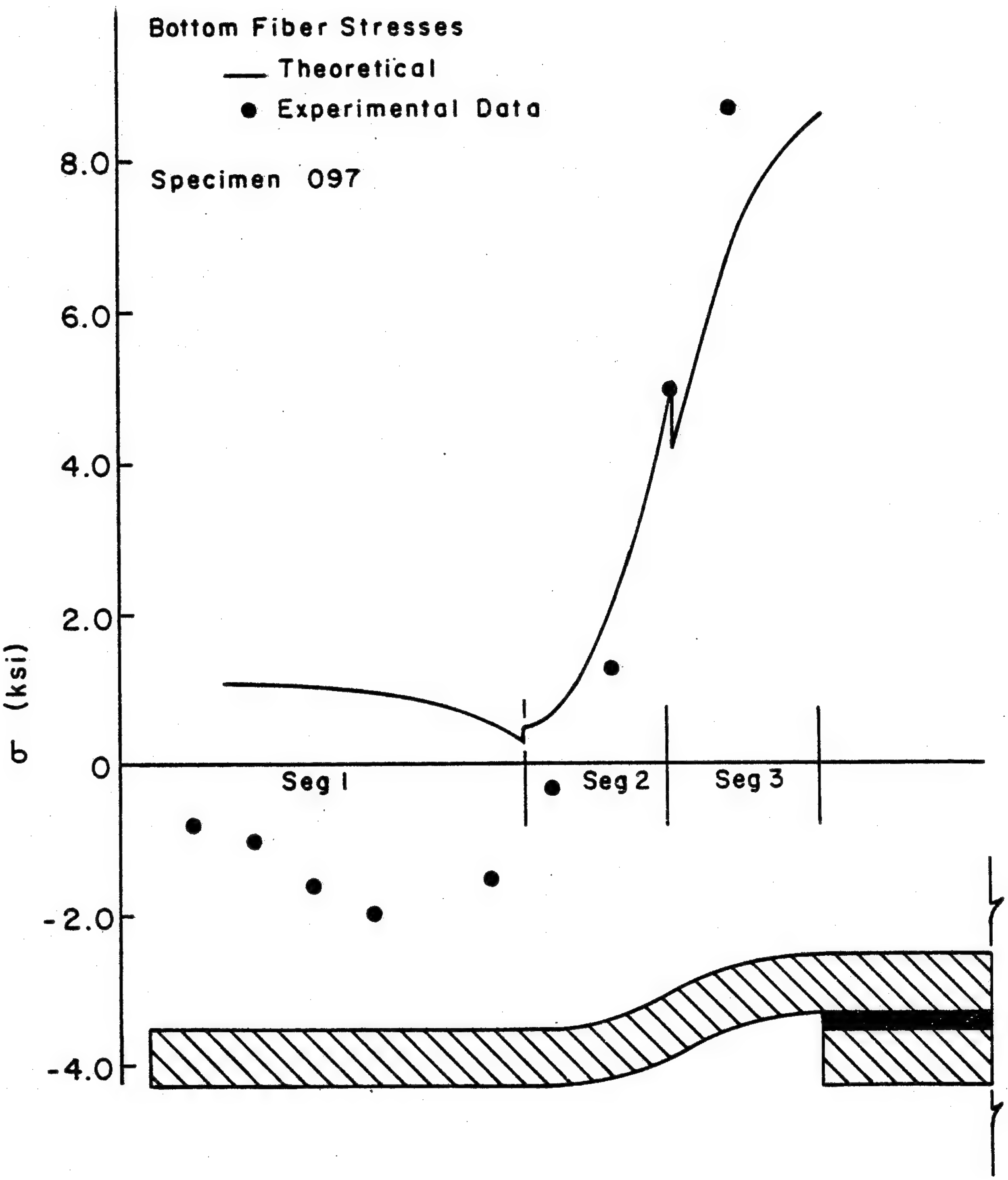

FIGURE 30: ELASTIC RESPONSE DUE TO TENSION - BOTTOM FIBER STRESSES

that the joint invariably strained beyond the small-deflection range at considerably small loadings. It was therefore a rather arduous task to approximately determine the experimentally applied moment to the joint. The correlation between the theoretical and experimental data may be referenced in Figures 31-34. As in the case of tensile loading, it should be noted that the stresses in SEG1 are again considerably higher than those predicted by theory, which is attributable to the molded geometry.

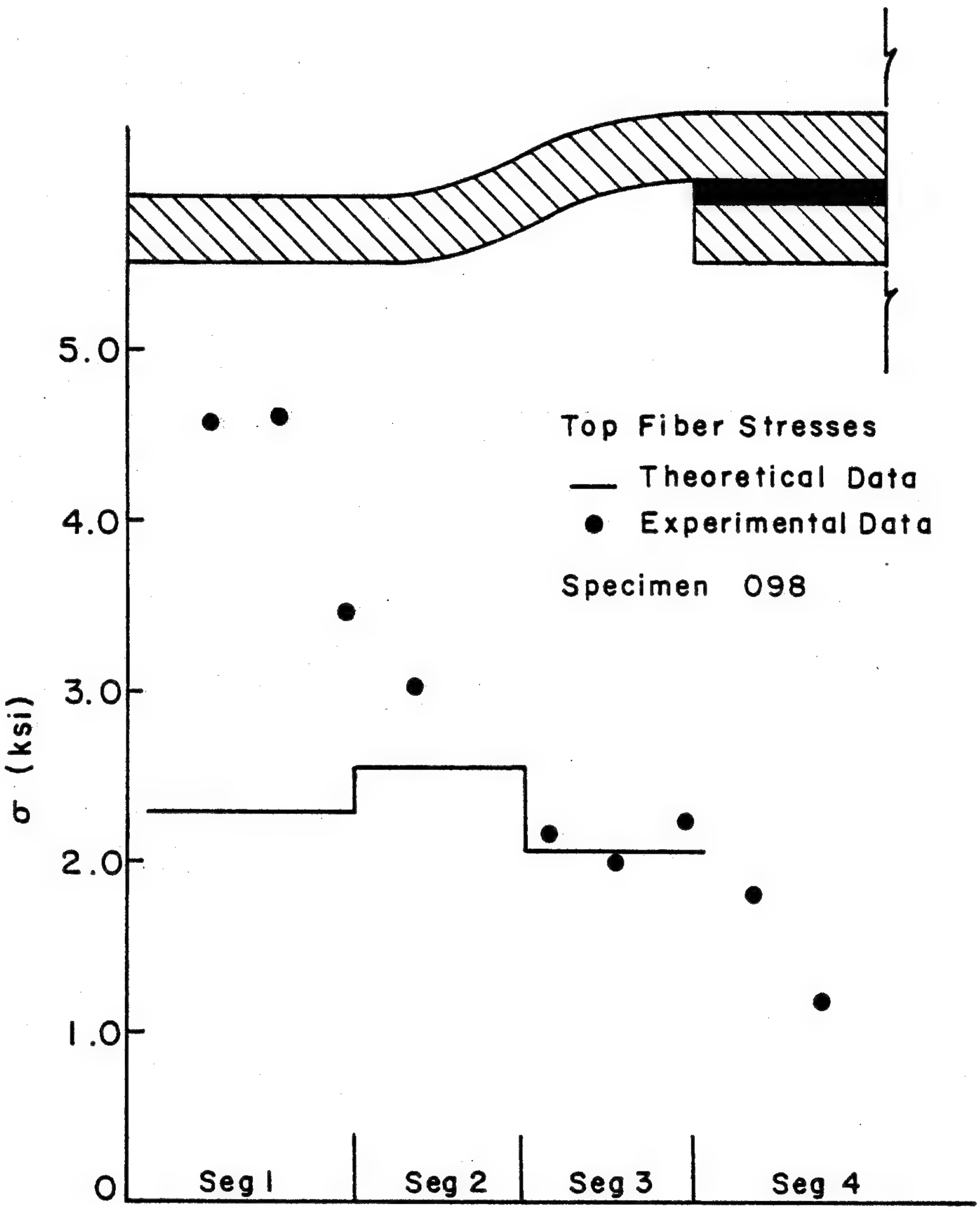

FIGURE 31: TOP FIBER STRESSES DUE TO BENDING

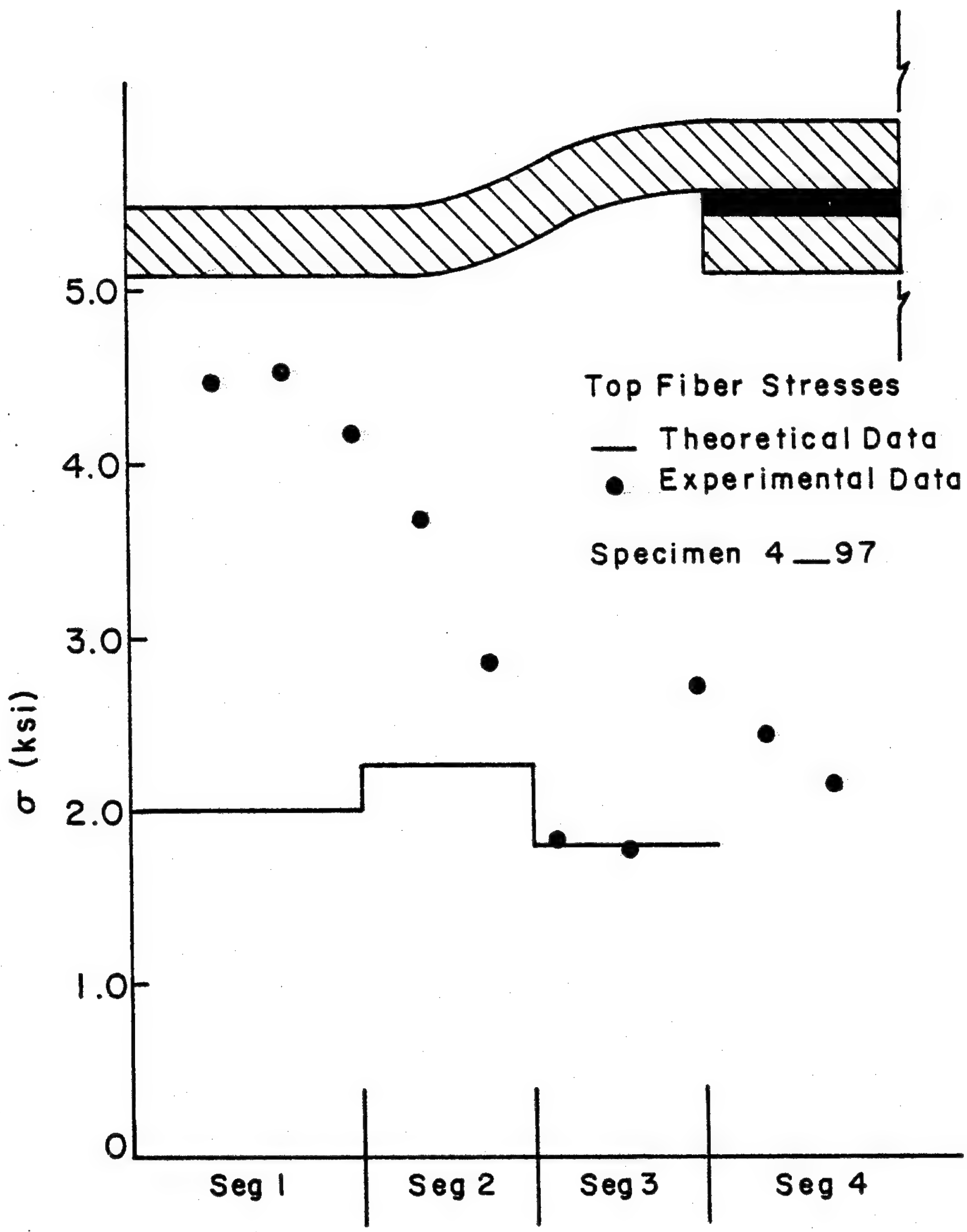

FIGURE 32: TOP FIBER STRESSES DUE TO BENDING

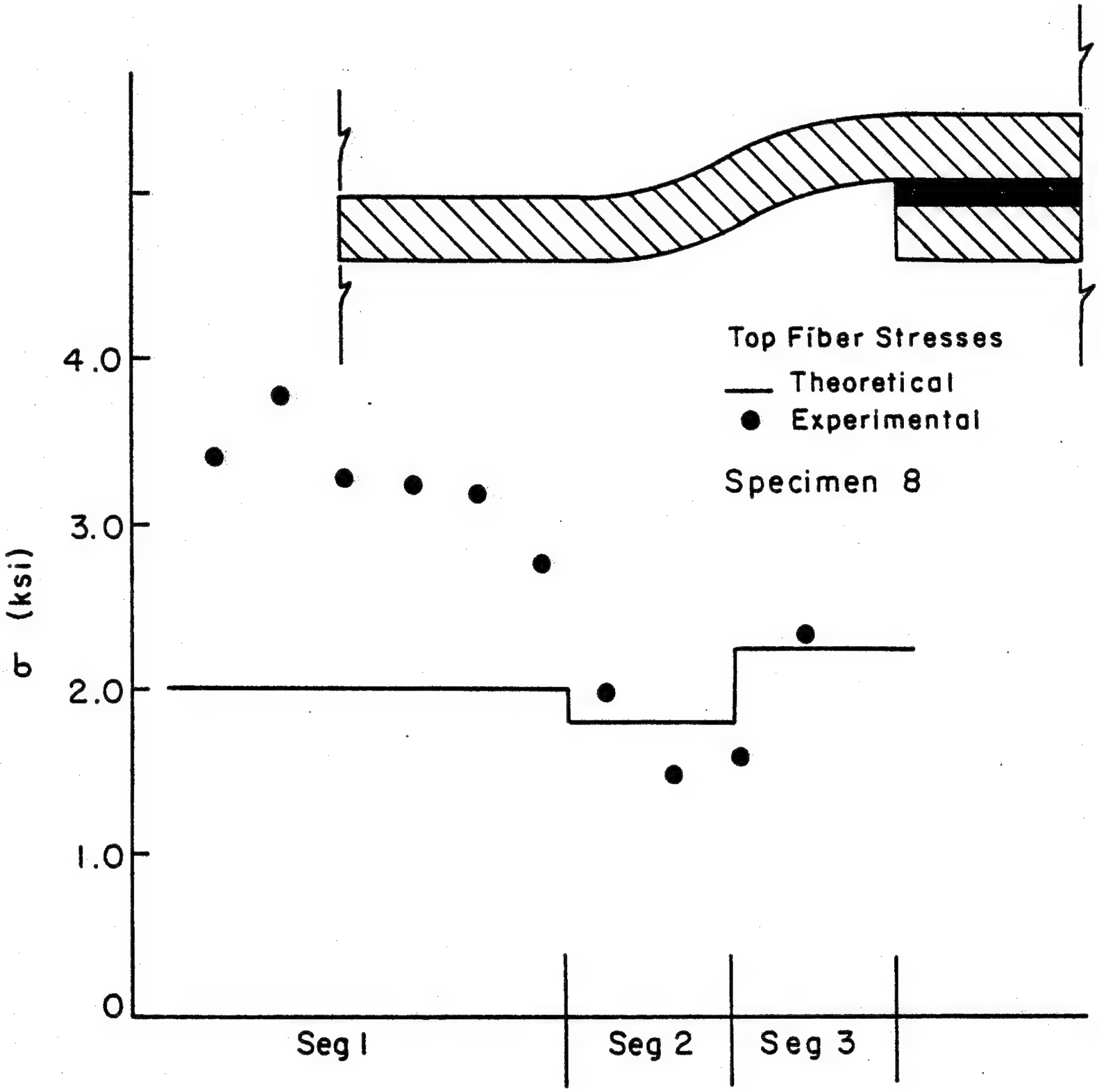

FIGURE 33: BOTTOM FIBER STRESSES DUE TO BENDING

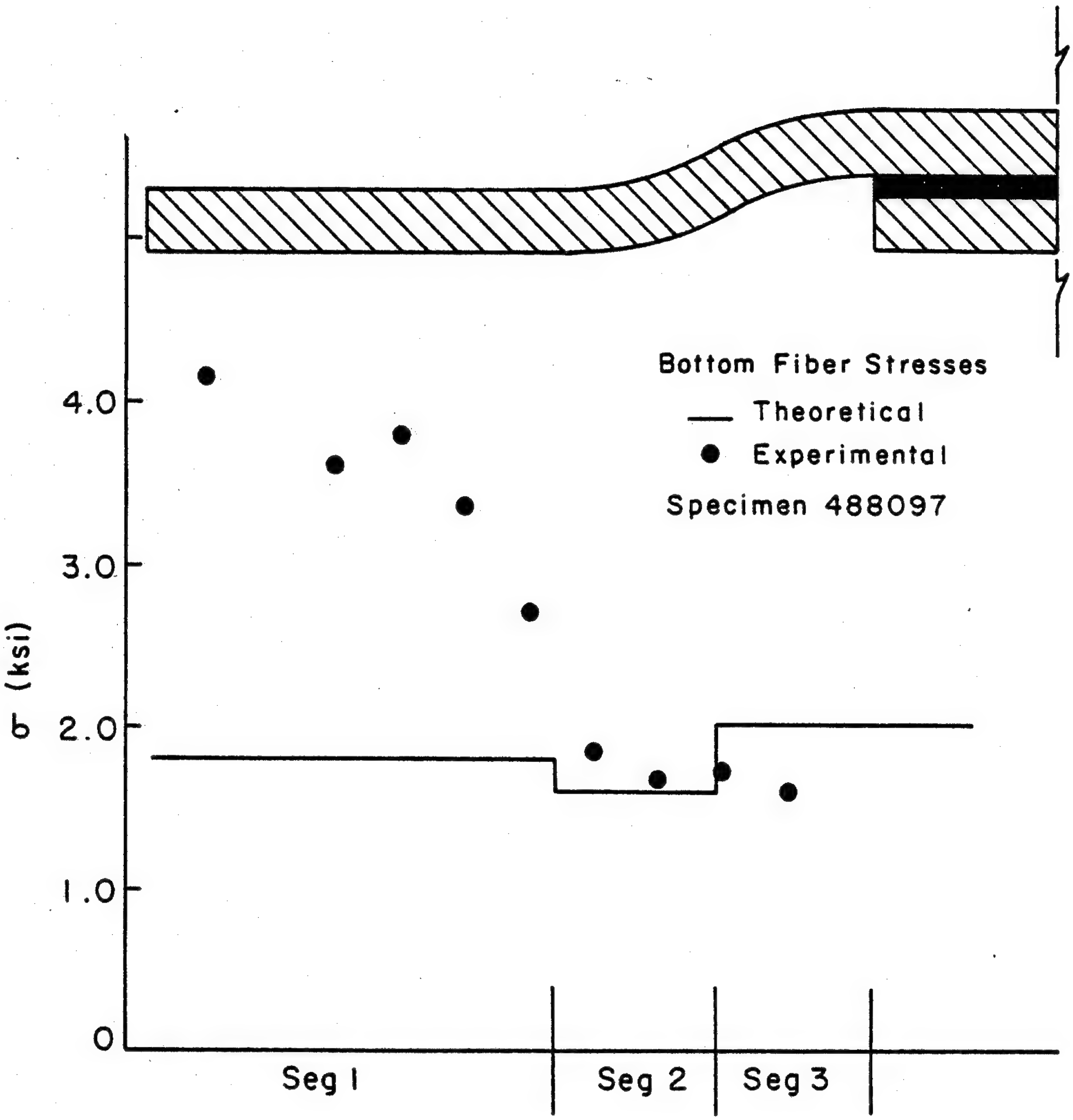

FIGURE 34: BOTTOM FIBER STRESSES DUE TO BENDING

Table 3

Experimental Results from Tension Tests

Specimen #	Contact Area (in^2)	Loading Condition	Failure Load (LBS)	Failure Mode*
4__097	1	tension	191	flexure
4_8097	1	tension	206	flexure
4___98	1	tension	209	flexure
4__098	1	tension	171	flexure
___97	1.75	tension	169	flexure
___98	1.75	tension	242	flexure
__097	1.75	tension	151	flexure
_8098	1.75	tension	200	flexure

*after failure initiation, it was observed that the crack was propagated via interlaminar shear

V. Failure Analysis

One of the most important parameters to predict in a study of this type is the ultimate loading conditions. This in essence dictates the choice of a failure criterion. The maximum stress theory will be employed in this report because of its simplicity in application and execution. Other popular failure criteria, such as the Tsai-Wu criterion were deemed inappropriate due to the limiting assumptions made in accordance with beam theory.

Maximum stress criterion states that the material will fail when any component of stress exceeds the corresponding material strength. In general, the above statement may be written in equation form as

$$\sigma_i \geq X_i^T \qquad (\sigma_i > 0) \qquad i = 1\text{-}3 \tag{14}$$

$$|\sigma_i| \geq X_i^C \qquad (\sigma_i < 0) \qquad i = 1\text{-}3 \tag{15}$$

$$|\sigma_i| \geq S_i \qquad i = 4\text{-}6 \tag{16}$$

where X_i^T = ultimate tensile strength

X_i^C = ultimate compressive strength

S_i = ultimate shear strength

These equations simplify to those listed below after employing the local coordinate nomenclature for the

"joggle-lap" joint.

$$\sigma_u \geq X^T \qquad (\sigma_u > 0) \tag{17}$$

$$|\sigma_u| \geq X^C \qquad (\sigma_u < 0) \tag{18}$$

$$|\sigma_{us}| \geq S_i \tag{19}$$

Applying this failure criterion to the model, it was found that the bottom fiber tensile stresses (see Figure 35) predicted the ultimate loading of the joint within experimental error. Thus the maximum flexural stress was utilized to predict failure.

All failures occurring as a result of tensile loading were initiated along the bottom surface of SEG3. Crack initiation was observed to be of the net tension mode, while propagation appeared to be due to "interlaminar shear". There was a general consistency among the initiation and propagation of the crack for all tension tests.

It was thought at one time that the curved sections of the joint (SEG2, SEG3) were either fiber deficient or highly anisotropic yielding a potential low strength area. However, a photomicrograph of this cross-sectional area clearly shows no such tendencies. (See Plate 7)

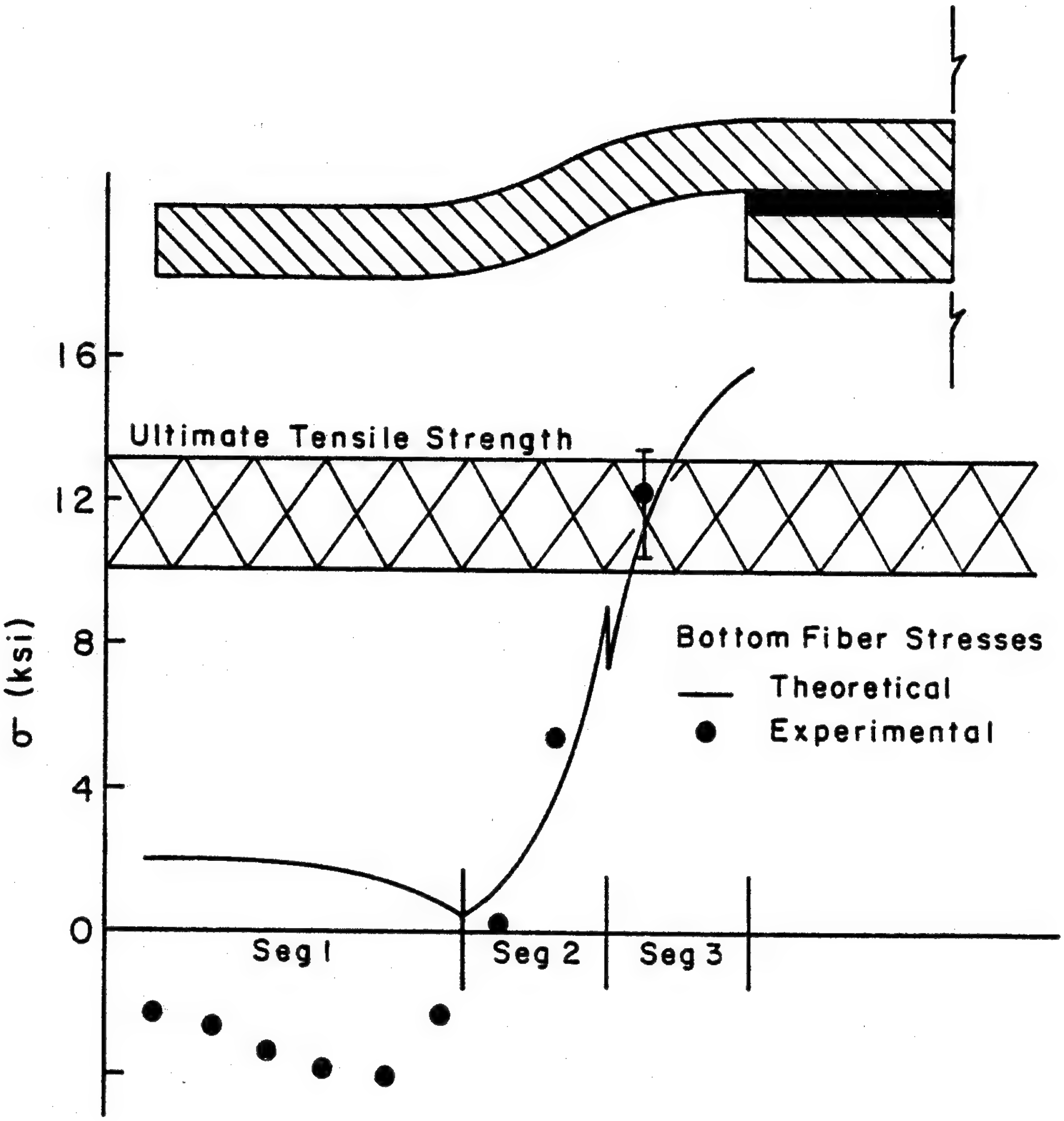

FIGURE 35: BOTTOM FIBER STRESSES AT THE FAILURE LOAD

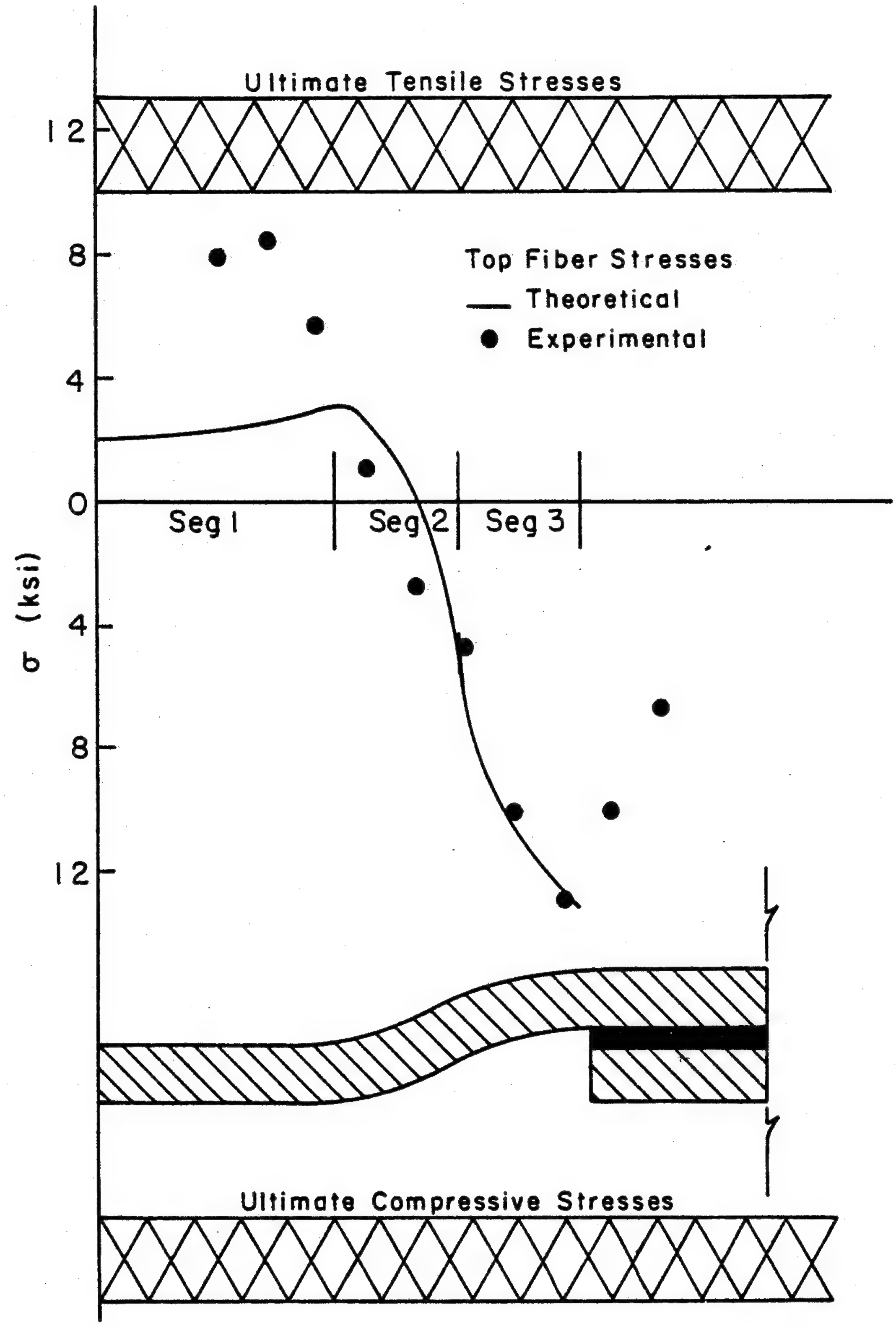

FIGURE 36: TOP FIBER STRESSES AT THE FAILURE LOAD

VI. Conclusions

The response of the "joggle-lap" joint was investigated for both tensile and bending loads in this report. It was found that experimental data correlated rather well to the values of stress predicted by the analytical model. The results of the bending study were not as favorable, in that experimental verification proved to be more difficult.

A parametric study was undertaken for the "joggle-lap" joint subject to tensile loads in an effort to isolate the crucial design parameters. In Figures 37 through 40 a normalized stress value is plotted against one of four parameters - adherent thickness, inside radius, contact area, and load. From these design curves the following conclusions are inferred.

- If weight saving requirements are not stringent, the effect of increasing adherent thickness drastically reduces maximum flexural adherent stress.
- Increasing the radius of curvature has a negligible effect on reducing maximum adherent stress due to a trade-off between mechanisms.

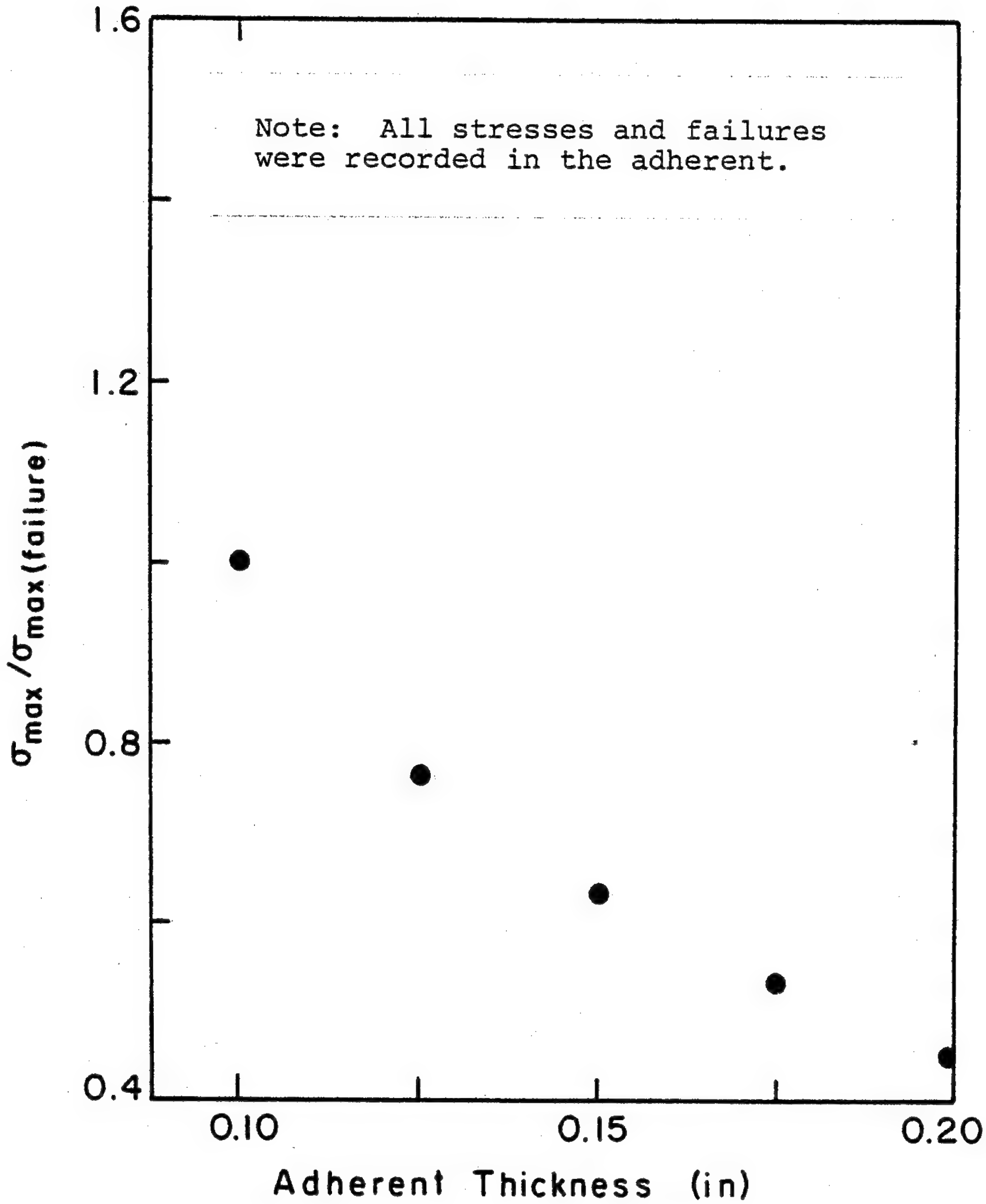

FIGURE 37: EFFECTS OF ADHERENT THICKNESS ON JOINT STRENGTH

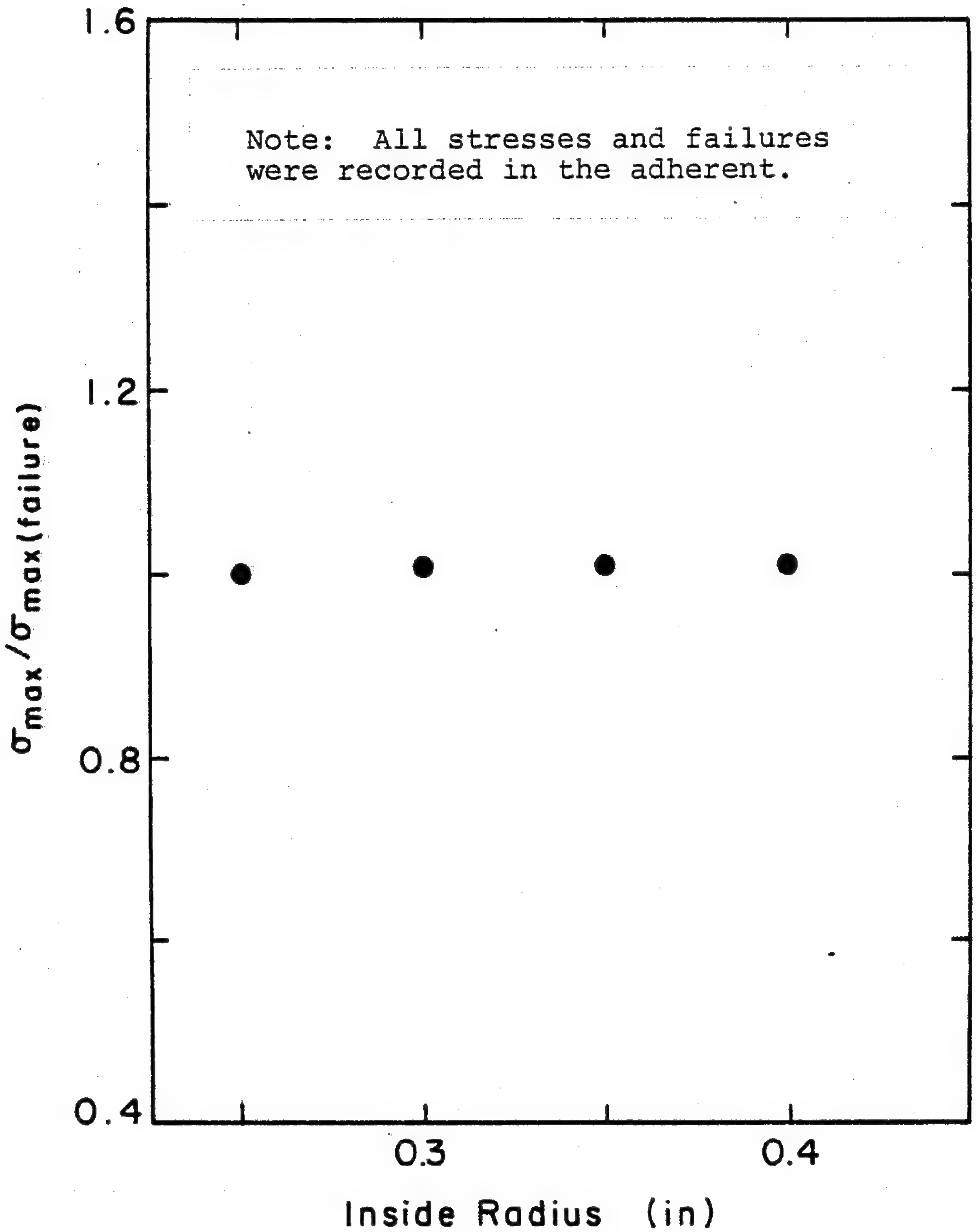

FIGURE 38: EFFECTS OF INSIDE RADIUS OF JOINT STRENGTH

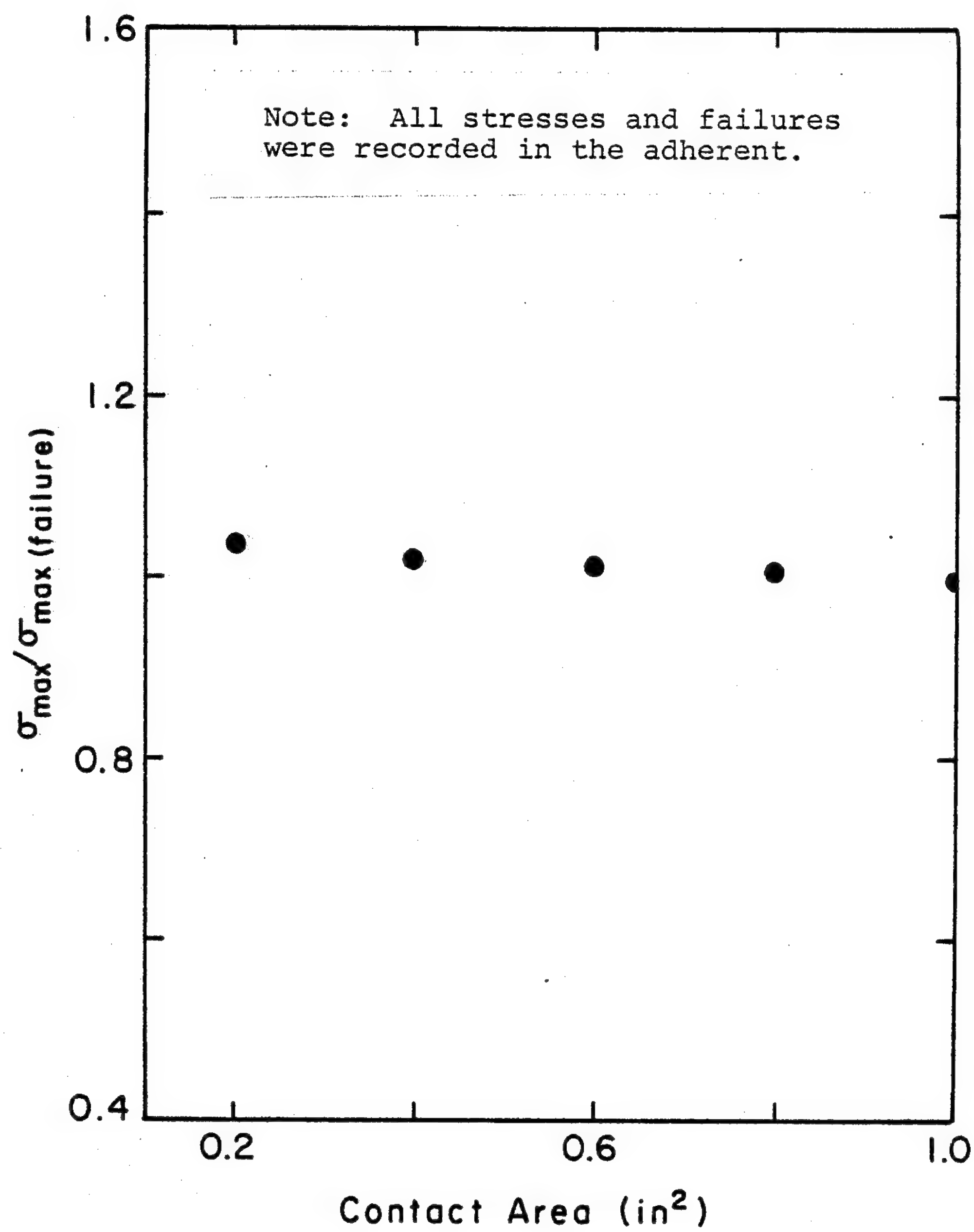

FIGURE 39: EFFECTS OF CONTACT AREA ON JOINT STRENGTH

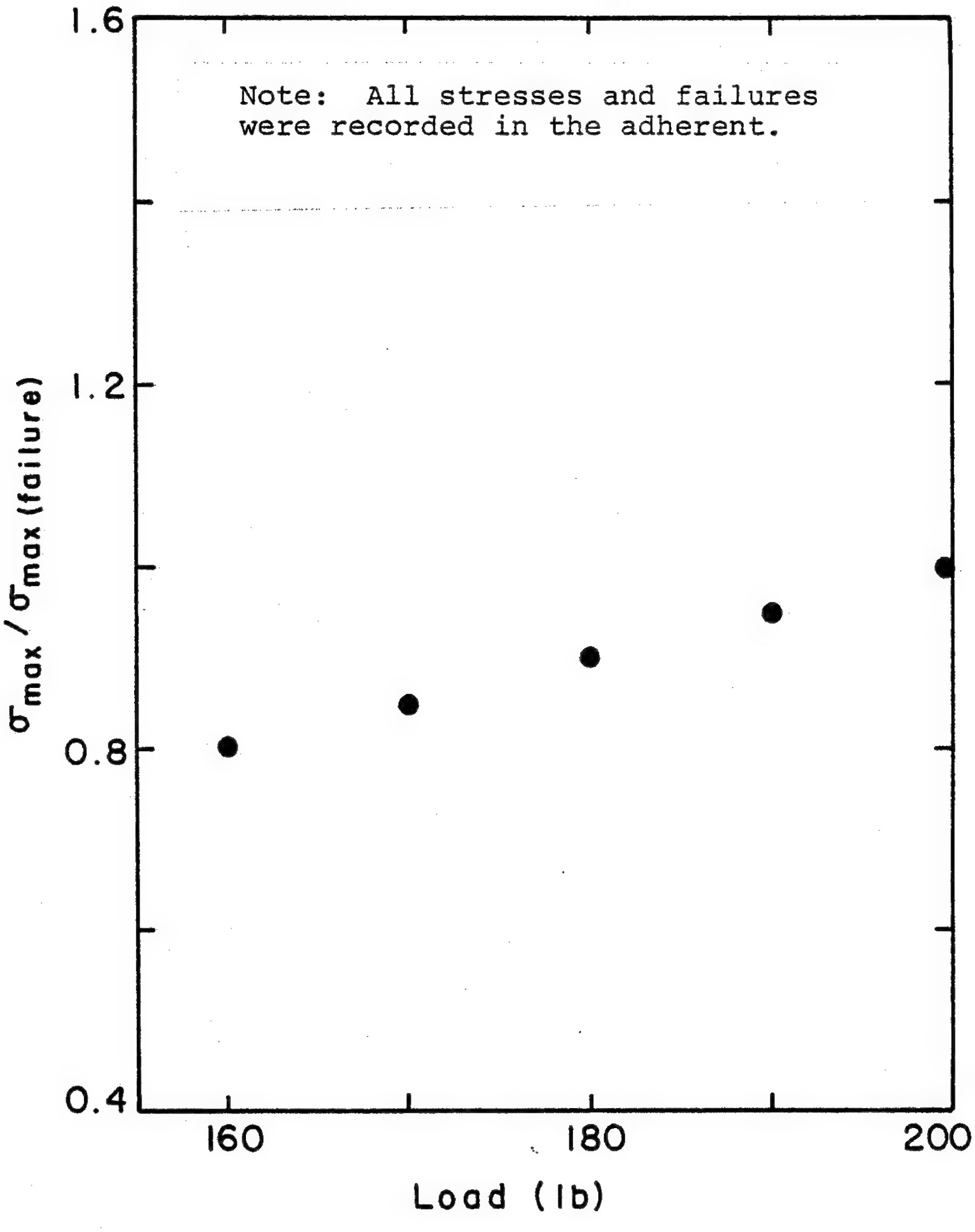

FIGURE 40: EFFECT OF LOAD ON JOINT STRENGTH

- Neglecting local stress concentrations, the effect of reducing the overlap length does not increase adherent stress significantly.

- In the region of the failure load, the maximum adherent stress increases linearly with load.

An important parameter in joint design is that of joint efficiency. This parameter is defined to be the ratio of ultimate joint load divided by the ultimate load carried by the material if the joint were not present. The joint efficiency of the "joggle-lap" joint in tension is calculated to be 0.153.

The adhesive system employed in this report proved to be quite adequate from a structural point of view. For the given overlap length of 1 in (2.54 cm) there were no recorded failures in the adhesive layer. Failure loads were predicted using the maximum flexural stress as the limiting criterion.

This report would be incomplete if it did not offer several suggestions for future work as an outgrowth of this study. An obvious limitation to the work reported herein is the inability to extensively verify the analytical model by experimental testing of various joint geometries.

Further development in this area would greatly increase the reliability of the computer model.

More detailed work needs to be completed in the response of the "joggle-lap" joint to bending loads. This report included only a cursory investigation of bending behavior as a means of identifying the underlying problems associated with the experimental verification of theory.

It is felt that this report will provide a fundamental basis for future research concerning the "joggle-lap" joint.

VII. Acknowledgements

The authors wish to thank David W. Adkins and Joseph J. Quigley, graduate students at the University of Delaware, for their expertise and guidance throughout this research effort. Also, we wish to express our appreciation to Dr. Terry V. Baughn and Bill Englehart of International Harvester for their vested interest in the program and for supplying all of the test specimens. Special thanks are also directed to Larry Carapellotti and his staff at Goodyear Adhesives for their assistance in bonding the experimental specimens.

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X. Appendices

Appendix A

Derivation of the Governing Equations
for a Curved Beam

Consider the curved beam element shown in Figure 41.

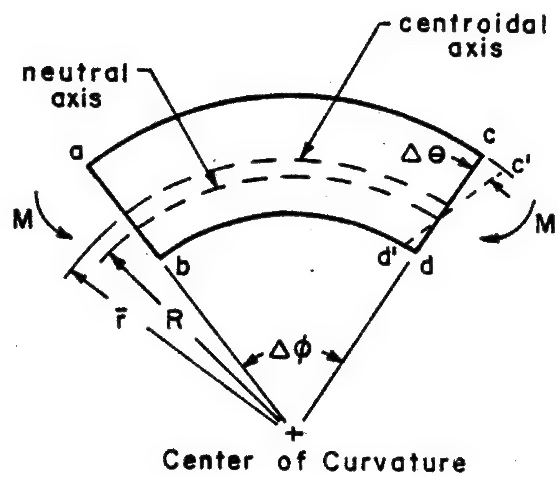

FIGURE 41: CURVED BEAM ELEMENT

The analysis begins by seeking an expression for the strain distribution perpendicular to the neutral axis. Assume that the curved beam, with an initial radius of curvature R, undergoes a small elastic deformation due to the applied moment. (It is important to note that the neutral axis of bending for a curved beam does not necessarily coincide with the centroidal axis of the beam.) Under the action of this moment it becomes apparent that segment cd rotates about the neutral axis

to a new position c'd'. It is assumed here, as in classical beam analysis, that plane sections remain plane. It is readily seen that while the deformation of the beam varies linearly with the distance from the neutral axis, the strains do not. The reason is that the original length of all the fibers prior to the application of the moment are not constant.

Thus the following relation for the strain distribution is written below.

$$\varepsilon = \frac{e_\ell}{\Delta_L} = \frac{-y\Delta\theta}{(R-y)\Delta\phi} \tag{20}$$

where e_ℓ = elongation

y = radial coordinate (positive radially inward)

$\Delta\theta$ = angle of deformation

$\Delta\phi$ = angle subtended by curved beam

The above equation shows the strain to vary hyperbolically across the section. Using the plane stress constitutive relation, Eq. (20) becomes

$$\sigma = \frac{-Ey\ \Delta\theta}{(R-y)\Delta\phi} \tag{21}$$

Now it is appropriate to derive the formulas for flexural stress. First assume that the portion of the beam is in equilibrium. Following directly one may write the equations of equilibrium for an arbitrary section.

$$\Sigma F_{axial} = 0$$

$$\int_A \sigma da = 0 \tag{22}$$

Making the appropriate substitutions for the stress Eq. (22) becomes

$$\int_A \frac{-Ey\Delta\theta da}{(R-y)\Delta\phi} = 0 \tag{23}$$

Assuming E, $\Delta\phi$, and $\Delta\theta$ to be constants the integral is simplified as shown in Eq. (24).

$$\int_A \frac{yda}{(R-y)} = 0 \tag{24}$$

It is possible to solve Eq. (24) for the radius of curvature and thus locate the neutral surface; however, it will suffice to let Eq. (24) stand as is for now.

Referring to Figure 42 and summing moments about the

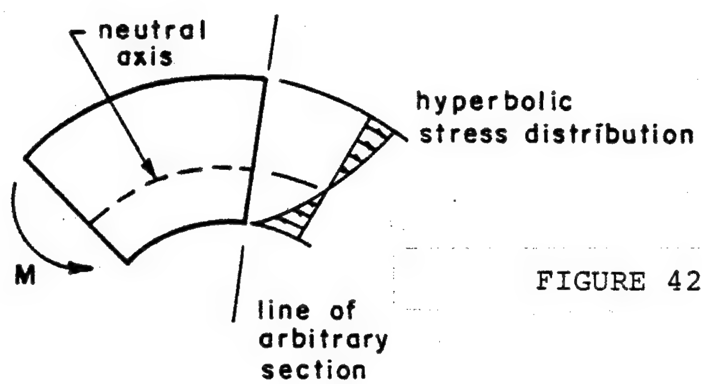

FIGURE 42

neutral axis, one finds that the stress distribution must also satisfy the equation below.

$$M = -\int_A \sigma \, y da \tag{25}$$

making the appropriate substitutions, Eq. (25) becomes

$$M = -\int_A E\left(\frac{-y\Delta\theta}{(R-y)\Delta\phi}\right) y da \tag{26}$$

$$M = \frac{E\Delta\theta}{\Delta\phi} \int_A \frac{y^2}{(R-y)} da \tag{27}$$

Notice the algebraic relation that permits the substitution of an equivalent expression into Eq. (27).

$$\frac{y^2}{R-y} = \frac{Ry}{R-y} - y \tag{28}$$

Eq. (27) now becomes

$$M = \frac{E\Delta\theta}{\Delta\phi}\left[\int_A \frac{Ryda}{R-y} - \int_A yda\right] \tag{29}$$

and from the result of Eq. (24)

$$M = \frac{E\Delta\theta}{\Delta\phi} \left(R(0) - a\bar{u}\right) \tag{30}$$

where a = area

$\bar{u}$ = distance between the neutral and centroidal axes

Rearranging Eq. (30) yields

$$\frac{\Delta\theta}{\Delta\phi} = \frac{-M}{Ea\bar{u}} \tag{31}$$

Comparing this equation with the well-known deflection equation for straight beams, it is apparent that

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{32}$$

the left hand side of Eq. (31) is not yet suitable. The ultimate goal of such an analysis is to seek an equation that relates the deflection of the neutral axis to the position along the neutral axis.

Consider Figure 43 shown below.

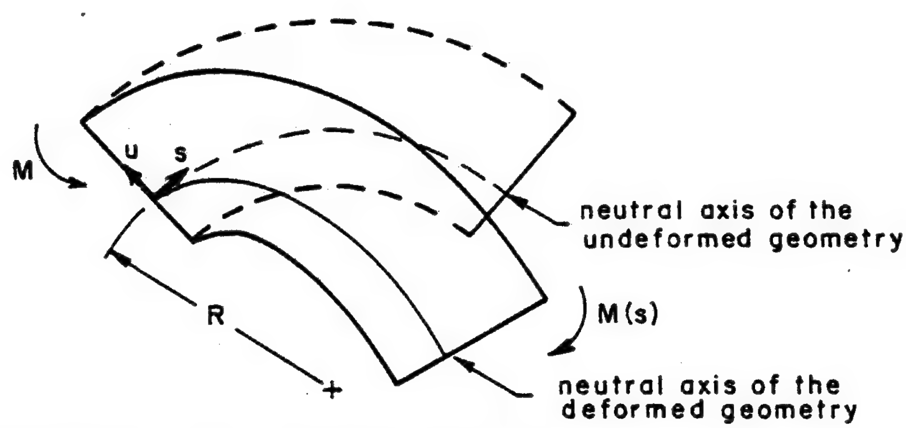

FIGURE 43: CURVED BEAM ELEMENT SUBJECT TO DEFLECTION

The beam is deflected as shown to illustrate the most general case of a non-constant moment. That is, the moment is a function of position. Now the deflection can be measured as the deviation between the undeformed neutral surface and the deformed neutral surface. For convenience just the neutral axis and appropriate parameters are drawn in Figure 44.

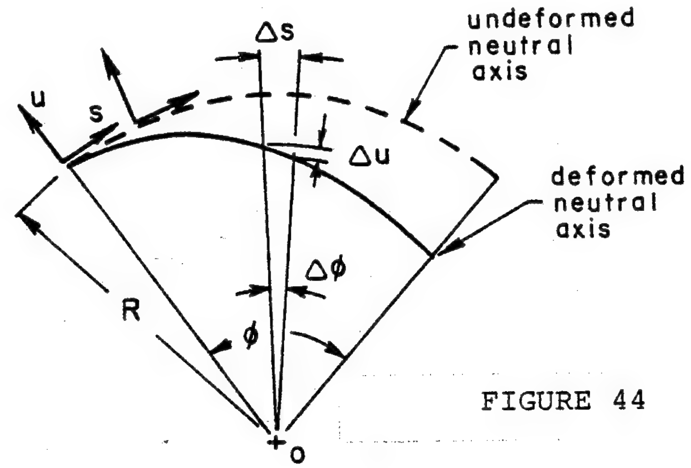

FIGURE 44

A coordinate system u,s is defined and shown in the figure where s traverses tangentially to the undeformed neutral axis and u is defined to be perpendicular to that axis.

Enlarging the area of interest and focusing on the triangle of Figure 45, one finds that

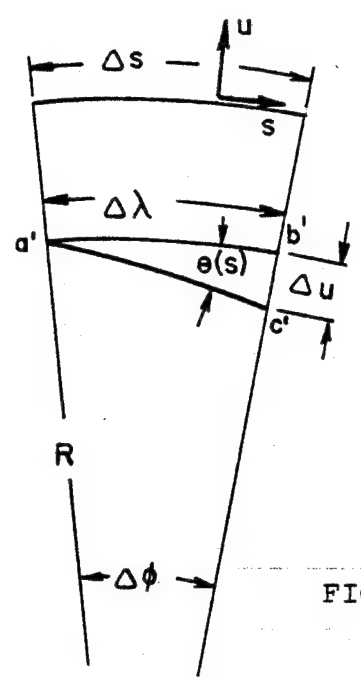

FIGURE 45

$$\theta(s) = \frac{\Delta u}{\Delta \lambda}$$

Realizing that tan $\alpha \cong \alpha$ for small α, it follows that

$$\Delta\lambda = \frac{(R+u)\Delta S}{R}$$

and thus

$$\underline{\theta(s)} = \frac{R\Delta u}{(R+u)\Delta s}$$

which may be written as

$$\frac{\Delta u}{\Delta s} = \frac{(R+u)\,\theta(s)}{R} \qquad (33)$$

Finally in the limit as $\Delta s \to 0$: Eq. (33) becomes

$$\lim_{\Delta s \to 0} \frac{\Delta u}{\Delta s} = \frac{(R+u)\theta(s)}{R} = \frac{du}{ds} \qquad (34)$$

From Eq. (31), several simplifications can be made with the proper substitutions.

$$\frac{\Delta\theta}{\Delta\theta} = \frac{-M}{Ea\bar{u}}$$

where $\Delta\phi = \frac{\Delta s}{R}$

$$\lim_{\Delta s \to 0} \frac{\Delta\theta}{\Delta s} = \frac{-M}{REa\bar{u}} = \frac{d\theta}{ds} \qquad (35)$$

Differentiating Eq. (34) with respect to s yields

$$\frac{d^2u}{ds^2} = \frac{(R+u)}{R}\frac{d\theta}{ds} \tag{36}$$

and substituting Eq. (35) into Eq. (36) yields the final results - a second order differential equation relating deflection to position in terms of the applied moment.

$$\frac{d^2u}{ds^2} = \frac{(R+u)M}{R^2Ea\bar{u}} \tag{37}$$

Appendix B

Beam Bending Model of the "Joggle-Lap" Joint

SEG1 may be modeled as a straight beam shown in Figure 46.

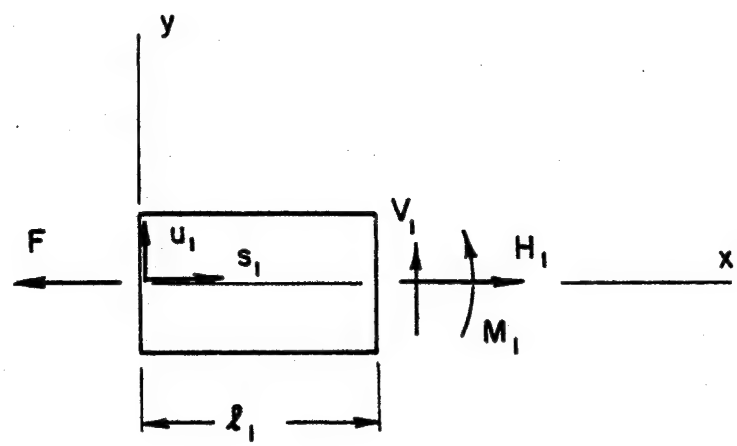

FIGURE 46: SEG1 MODELED AS A STRAIGHT BEAM

In general, the moment experienced by any segment originates from two sources: eccentricity from geometry and eccentricity due to deflection. The preceding statement may be written algebraically as follows.

$$M = F(e_{geom} + e_{defl}) \tag{38}$$

where M = moment

F = applied force

e = eccentricity

It is readily seen that $e_{geom} = 0$ for SEG1. Writing Eq. (38) in the local coordinate system, the moment experienced by

this segment reduces to

$$M = Fu_1 \tag{39}$$

where

u_1 = deflection in the local coordinate system

Substituting Eq. (39) into Eq. (5) yields

$$\frac{d^2u_1}{ds_1^2} - \frac{Fu_1}{EI} = 0 \tag{40}$$

The corresponding boundary conditions are expressed below

$$u_1(0) = 0$$
$$u_1(\ell_1) = u_o$$

where u_o is yet undetermined.

The solution of Eq. (40) is of standard form and known to be

$$u_1 = C_1 \sinh\sqrt{F/EI} \cdot s_1 + C_2 \cosh\sqrt{F/EI} \cdot s_1 \tag{41}$$

Applying the boundary conditions to Eq. (41) determines the constants C_1 and C_2 to be

$$C_2 = 0$$
$$C_1 = u_o \Big/ \sinh\sqrt{F/EI} \cdot \ell_1$$

and thus

$$u_1 = u_o \frac{\sinh\sqrt{F/EI} \cdot s_1}{\sinh\sqrt{F/EI} \cdot \ell_1} \qquad 0<s_1<\ell_1 \tag{42}$$

where u_o is necessarily negative to correspond with the physical system. In other words, for a given tensile load it is expected that SEG1 will deflect downward. (Figure 6). Also

$$\frac{du_1}{ds_1}(\ell_1) = u_o \sqrt{F/EI}\ \frac{\cosh\sqrt{F/EI}\ \ell_1}{\sinh\sqrt{F/EI}\ \ell_1}$$

It should be noted that the deflection as given by Eq. (42) is not known explicitly in terms of the given parameters. U_o is still unknown and it will be shown later how this value may be determined uniquely.

SEG2 is modeled as a curved beam and shown in Figure 47. The local coordinate system is a curvilinear coordinate system with the s_2 axis traversing the neutral axis as shown. Positive deflections are measured normal to the undeformed neutral axis in the direction of u_2.

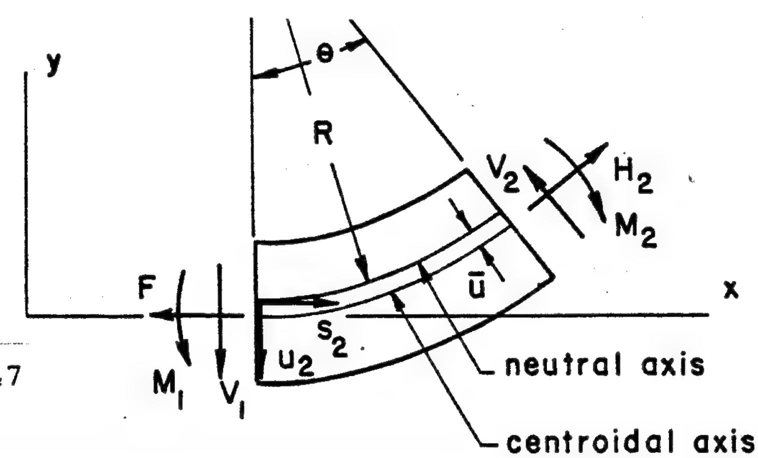

FIGURE 47

From the derivation of the general case for a curved beam in pure bending (see Appendix A), the governing equation for the deflection is

$$\frac{d^2u_2}{ds_2^2} = \frac{(R + u_2)M}{R^2Ea\bar{u}} \tag{43}$$

where

s_2 = arc length

u_2 = deflection normal to neutral axis

M = moment

R = radius of curvature

E = modulus of elasticity

a = cross sectional area

$\bar{u}$ = distance between neutral axis and centroidal axis and its value is necessarily negative

The moment may be written as the product of the applied load and the eccentricity, where the eccentricity in this case consists of both geometry and deflection considerations.

At this point, it is appropriate to introduce the notion of extensional effects. It is realized that with the given loading conditions, the "joggle-joint" will undergo deflections parallel to the neutral axis as well. This fact would be of little concern if all beam segments of the joint configuration had their neutral axis aligned with the loading axis. If this were the case, the longitudinal displacement would not affect the eccentricity.

However, it is evident that the extensional strains in the curved beam segments give rise to an added component of eccentricity defined to be e_{ext}. To calculate the value of e_{ext}, one merely applies the criterion of force equilibrium to SEG2 (Figure 47) in the local coordinate system.

$$\Sigma F_{u_2} = 0 \qquad H_2\cos\theta + V_2\sin\theta = F$$

$$\Sigma F_{s_2} = 0 \qquad H_2\sin\theta = V_2\cos\theta$$

thus $\quad H_2 = F\cos\theta$

where $\quad \theta$ = angle subtended by SEG2

Employing the constitutive relationship

$$\sigma = E\varepsilon$$

where $\quad \sigma$ = stress

E = modulus of elasticity

ε = strain

and considering only the y (global coordinate) component of the extension we thus arrive with the expression for e_{ext}.

$$e_{ext} = \frac{Fs_2\cos(s_2/R)\sin(s_2/R)}{aE} \tag{44}$$

Eq. (44) must be added to the other terms which comprise the eccentricity due to deflection.

Therefore Eq. (43) becomes

$$\frac{d^2u_2}{ds_2^2} = \frac{-(R-u_2)F}{R^2Ea\bar{u}}\left[e_{geom} + e_{defl} + e_{ext}\right] \tag{45}$$

where
$$e_{geom} = R(1-\cos(\frac{s_2}{R}) + \bar{u})$$
$$e_{defl} = u_2\cos(\frac{s_2}{R})$$
$$e_{ext} = Fs_2\cos\frac{(s_2/R)\sin(s_2/R)}{aE}$$

Initial conditions for SEG2 are found by matching deflection and slope at the 1-2 interface.

$$u_2(0) = u_\circ$$

$$\frac{du_2}{ds_2}(0) = u_\circ \frac{\sqrt{F/EI}\;\cosh\sqrt{F/EI}\;\ell_1}{\sinh\sqrt{F/EI}\;\ell_1}$$

Using a numerical integration routine to solve Eq. (45) the deflection u_2 may be marched out as a function of arc length s_2. A Runge-Kutta method based on Verners fifth and sixth order pair of formulas was used. An explanation of the integration routine DVERK may be referenced in Appendix C.

Figure 48 shows SEG3 modeled as a curved beam. From Eq. (37) the governing differential equation for a curved beam in pure bending is

$$\frac{d^2u_3}{ds_3^2} = \frac{(R+u_3)}{R^2Ea\bar{u}}M \tag{46}$$

where $M = F(e_{geom} + e_{defl} + e_{ext})$

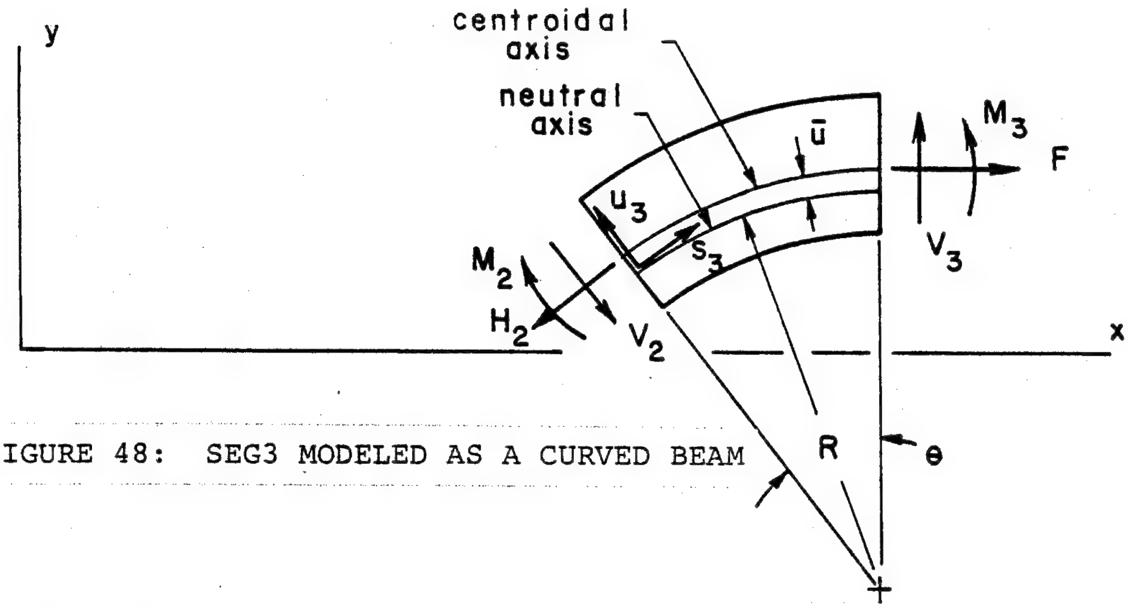

FIGURE 48: SEG3 MODELED AS A CURVED BEAM

Through geometric considerations e_{geom} can be shown to be

$$e_{geom} = \bar{u} + R(1-\cos\theta) - 2\bar{u}\cos\theta + R\sin(\frac{\pi}{2} - \theta + {}^{s}3/R) - \cos\theta \qquad (47)$$

where θ = angle subtended by SEG3

$\bar{u}$ = distance between centroidal and neutral axes

R = radius of curvature

s_3 = arc length along neutral surface of SEG3

Also $$e_{defl} = M\cos(\pi/2 - \theta + s3/R) \qquad (48)$$

From a similar argument developed earlier it may be shown that e_{ext} for SEG3 is given by

$$e_{ext} = \frac{Fs_3\cos(\theta-s_3/R)\ \sin\ (\theta-s_3/R)}{aE} \qquad (49)$$

Thus Eq. (46) becomes

$$\frac{d^2u_3}{ds_3^2} = \frac{-(R+u)}{R^2Ea\bar{y}} F\ (e_{geom} + e_{defl} + e_{ext}) \tag{50}$$

where e_{geom}, e_{defl}, and e_{ext}, are given by Eqs. (47), (48), and (49) respectively. Matching boundary conditions at the 2-3 interface provides initial conditions to Eq. (50) which may be integrated numerically as before.

SEG4 is analyzed as a multi-layered beam and shown in Figure 49. Treating this segment to be composed of three linear elastic beam elements, the governing differential equation follows from Eq. (5) with a slight modification.

$$\frac{d^2u_4}{ds_4^2} = \frac{M}{\sum_{i=1}^{3} E_iI_i} \tag{51}$$

where I_i = moment of intertia of the ith section about the neutral axis

E_i = modulus of elasticity of the ith element

$\sum_{i=1}^{3} E_iI_i$ is referred to as an effective flexural stiffness and is merely a constant. The moment is defined in the usual manner as

$$M = F(e_{geom} + e_{defl})$$

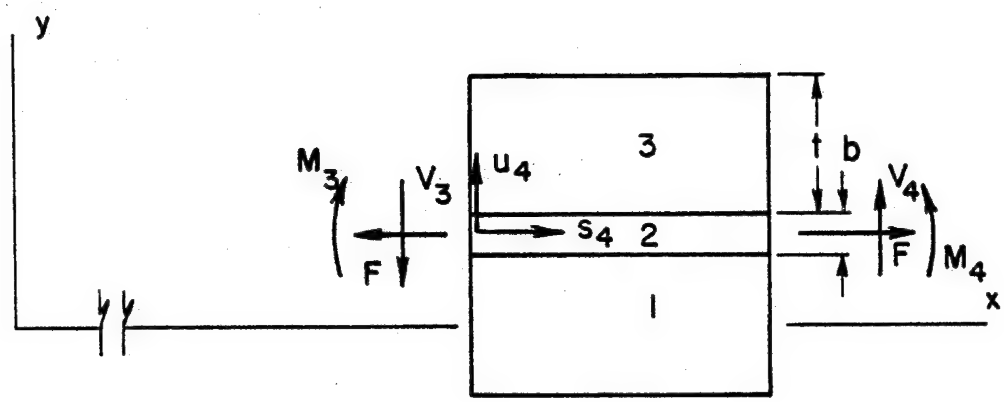

FIGURE 49: SEG4 MODELED AS A LAYERED BEAM

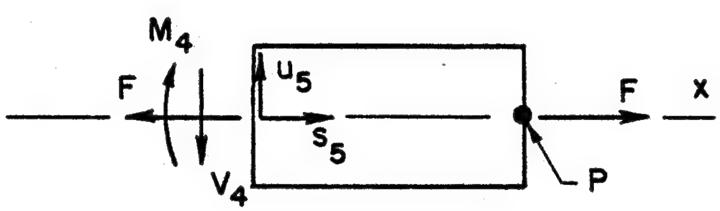

FIGURE 50: SEG5 MODELED AS A STRAIGHT BEAM

where $e_{geom} = .5(t+b)$

$e_{defl} = u_4$

Thus Eq. (51) becomes

$$\frac{d^2u_4}{ds_4^2} = \frac{F\ (.5(t+b) + u_4)}{\sum_{i=1}^{3} E_i I_i} \qquad (52)$$

Initial conditions are found by equating the deflection and slope at the 3-4 interface. Following in the usual manner, Eq. (52) is integrated to obtain an expression for the deflection of SEG4 as a function of arc length in the local coordinate system.

Finally SEG5 is shown in Figure 50 modeled as a straight beam member. The governing differential equation is the same as Eq. (5)

$$\frac{d^2u_5}{ds_5^2} = \frac{M}{EI} \qquad (53)$$

where $M = Fu_5$

and the initial conditions are obtained by matching the deflection and slope at the 4-5 interface. Upon integration of Eq. (53) the deflection SEG5 will be a known function of the abscissa s_5 of the local coordinate system. Therefore the deflection and slope at point P of Figure 50 are also known. But it should be apparent that the values of the deflection

and slope at this point must be zero or at least within certain tolerance limits. This in fact is the final boundary condition to the problem that is needed to uniquely determine the value of u_o which was previously assumed to be arbitrary. Thus, through an iterative process, a correct value of u_o may be calculated by assuring that the deflection and slope of point P of SEG5 is sufficiently[3] close to zero. To avoid confusion, it should be noted that by specifying zero deflection at point P we will force the slope to zero by the nature of the deflection function of SEG5. So in fact this is a well-posed problem, whereby we specify only enough boundary conditions as there are unknowns. The process for correctly determining u_o is shown schematically in Figure 51.

---

[3]it was found that reliable results were obtained by using the tolerance limits listed here.

$$|\text{deflection (P)}| < .00001$$
$$|\text{slope (P)}| < .00005$$

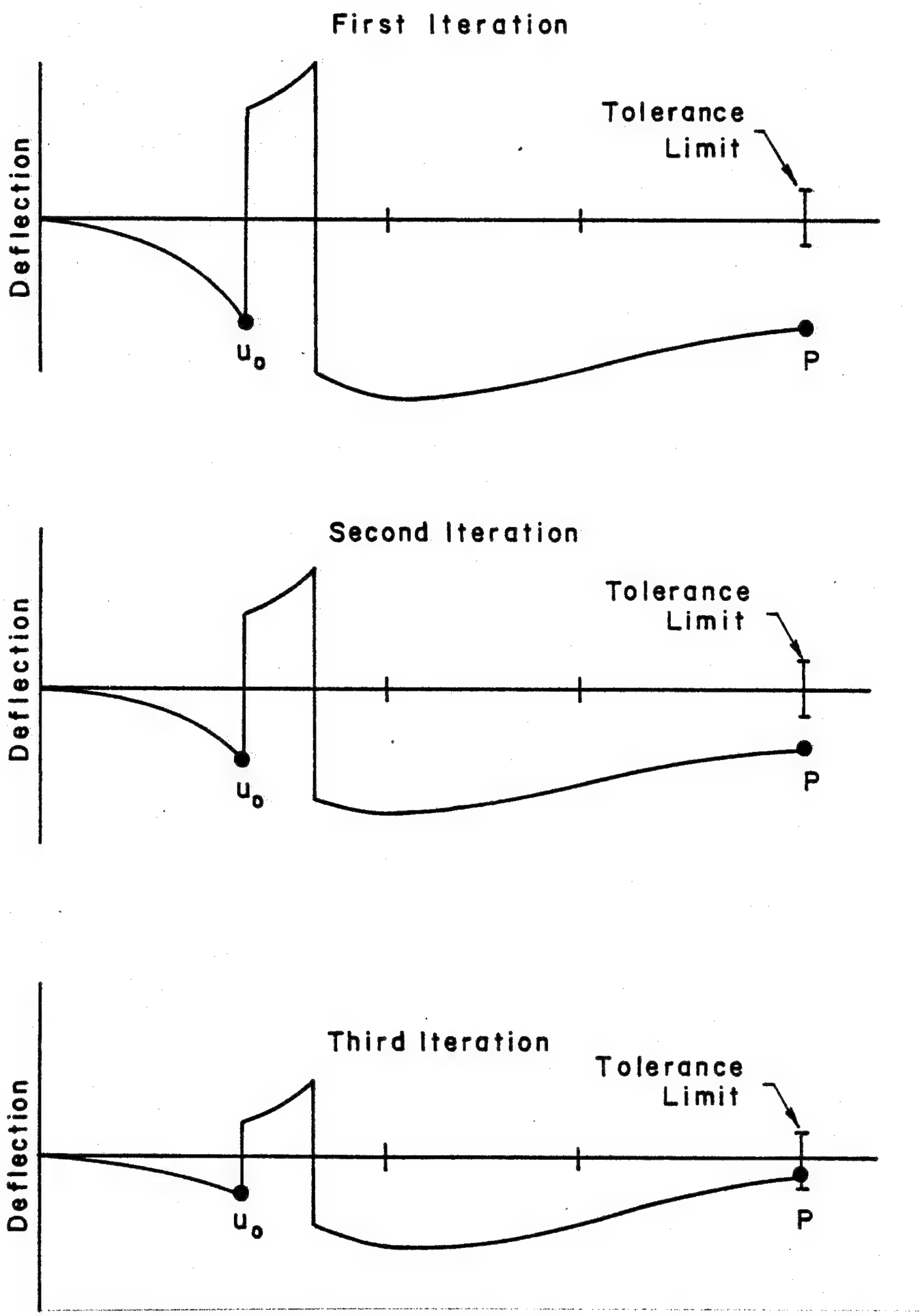

FIGURE 51: ITERATIVE PROCESS FOR DETERMINING u_0

Appendix C

Computer Programs

a. JOGGLE

To facilitate ease of calculation, a computer routine identified as JOGGLE was developed and may be referenced below. Essentially this program calculates a correct value of u_o and proceeds to determine a solution for the deflection while calculating stress profiles along the joint configuration. These stress profiles are linear in the straight beam members (SEG1, SEG5) and hyperbolic in the curved beams (SEG2, SEG3).

```
C-                         J    OOOOO    GGGGG    GGGGG    L         EEEEE
C-                         J    O   O    G        G        L         E
C-                         J    O   O    G  GG    G  GG    L         EEE
C-                     J   J    O   O    G   G    G   G    L         E
C-                     JJJJJ    OOOOO    GGGGG    GGGGG    LLLLL     EEEEE
C-
C-
C-
C-
C-
C-
C-                       ANALYTICAL BEAM BENDING MODEL
C-                             FOR A JOGGLE LAP JOINT
C-
C-
C-                    DEVELOPED BY:  RICHARD C. GIVLER
C-                         UNIVERSITY OF DELAWARE
C-                            SEPT 78 - JAN 79
C-
C-
C-
C-
$SET AUTOBIND
$RESET FREE
FILE  6(KIND=REMOTE.MAXRECSIZE=22)
      DIMENSION PROD(3). ERR(1). T(2).  EI(3).  YPRIME(2)
      COMMON R. PLOAD. ESMC. WIDTH. THICK. YBAR. THETA. EADH. PI
     -. BOND. J. TORC. TORCS. TRAC. TRACE. SHEAR
C-
C-
C-----PARAMETERS AND NOMENCLATURE
C-
C-
C-----MATERIAL THICKNESS
      THICK=.1
C-----LONGITUDINAL MODULUS OF ELASTICITY OF SMC IN PSI
      ESMC=2.1E+06
C-----MODULUS OF ELASTICITY OF ADHESIVE IN PSI
      EADH=1.0E+05
C-----LOAD IN LBS
      PLOAD=200.
C-----INSIDE RADIUS IN INCHES
      RADI=2.5*THICK
C-----OUTSIDE RADIUS IN INCHES
      RADO=3.5*THICK
C-----BONDING THICKNESS IN INCHES
      BOND=.03
C-----CONTACT WIDTH IN INCHES
      CONTA=1.0
C-----SPECIMEN WIDTH
      WIDTH=1.0
C-----LENGTH OF SEGMENT 1
      SEGA=3.5
C-----LENGTH OF SEGMENT 5
      SEGB=4.0
C-----LEFT INTERVAL LIMIT FOR ITERATION
      AINT=-.04
C-----RIGHT INTERVAL LIMIT FOR ITERATION
      BINT=+.01
C-----TOLERANCE LIMIT ON INITIAL DISPLACEMENT
      ERR(1)=.00001
```

```
C-----STEP SIZE FOR NUMERICAL INTEGRATION
      STEP=50.
      PI=3.141592654
C-----TRACING CONSTANTS
      TRAC=1.0
      TRACE=1.0
C-----
C-----NOTE:DEFLECTIONS ARE MEASURED NORMAL TO THE UNDEFORMED
C-      NEUTRAL AXIS
C-
      YO=(AINT+BINT)/2.
C-
C-----ITERATIVE CALCULATION OF YO TO FORCE ZERO DEFLECTION
C-----AND SLOPE AT END OF SEG5
C-----YO MUST LIE BETWEEN THE PROPOSED LIMITS OF AINT AND BINT
C-
C-*************************************************************
C-*************************************************************
C-*************************************************************
C NUMERICAL INTEGRATION VIA LIBRARY ROUTINES DVERK AND UERTST
C-*************************************************************
C-*************************************************************
C-*************************************************************
C-
 101  CONTINUE
      DO 200 X=0,SEGA+.005,SEGA/STEP
      PROD(3)=SQRT(PLOAD/(ESMC*(WIDTH*THICK**3./12.)))
      PROD(1)=YO*(.5*(EXP(PROD(3)*X)-EXP(-PROD(3)*X)))
      PROD(2)=.5*(EXP(PROD(3)*SEGA)-EXP(-PROD(3)*SEGA))
      T(1)=PROD(1)/PROD(2)
      T(2)=YO*PROD(3)/PROD(2)*(.5*(EXP(PROD(3)*X)+EXP
     -(-PROD(3)*X)))
 200  CONTINUE
      DIMENSION  C(24), Y(2),  W(2,9)
      EXTERNAL FCN1
C-----CALCULATION OF THE RADIUS OF CURVATURE FOR CURVED MEMBERS
      R=WIDTH*THICK/(ALOG(RADO/RADI))
      NW=2
C-----CALCULATION OF THE ANGLE SUBTENDED BY SEG2 AND SEG3
      THETA=ARCOS((5.*THICK-BOND)/(6.*THICK))
      N=2
C-----CALCULATION OF THE DISTANCE BETWEEN NEUTRAL AXIS AND
C-----CENTROIDAL AXIS OF CURVED MEMBERS
      YBAR=RADI+THICK/2.-R
      X=0.0
      Y(1)=-T(1)
      Y(2)=-T(2)
      TOL=.000001
      IND=1
      DO 300 Z=0.0,R*THETA+.001,R*THETA/STEP
      XEND=FLOAT(Z)
      CALL DVERK(N,FCN1,X,Y,XEND,TOL,IND,C,NW,W,IER)
      IF(IND.LT.0.OR.IER.GT.0) GO TO 20
 300  CONTINUE
  20  CONTINUE
      RINT=RNEW
      EXTERNAL FCN2
      X=0.0
      Y(1)=-Y(1)
      Y(2)=-Y(2)
      NW=2
      N=2
```

```
      IND=1
      DO 250 M=1,24
      C(M)=0.0
250   CONTINUE
      DO 350 Z=0.0,R*THETA+.001,R*THETA/STEP
      XEND=FLOAT(Z)
      CALL DVERK(N,FCN2,X,Y,XEND,TOL,IND,C,NW,W,IER)
      IF(IND.LT.0.OR.IER.GT.0) GO TO 70
350   CONTINUE
 70   CONTINUE
      EXTERNAL FCN3
      X=0.0
      NW=2
      N=2
      IND=1
      DO 290 M=1,24
      C(M)=0.0
290   CONTINUE
      DO 400 Z=0.0,CONTA+.005,CONTA/STEP
      XEND=FLOAT(Z)
      CALL DVERK(N,FCN3,X,Y,XEND,TOL,IND,C,NW,W,IER)
      IF(IND.LT.0.OR.IER.GT.0) GO TO 80
400   CONTINUE
 80   CONTINUE
      EXTERNAL FCN4
      X=0.0
      NW=2
      N=2
      IND=1
      DO 291 M=1,24
      C(M)=0.0
291   CONTINUE
      DO 246 Z=0.0,SEGB+.005,SEGB/STEP
      XEND=FLOAT(Z)
      CALL DVERK(N,FCN4,X,Y,XEND,TOL,IND,C,NW,W,IER)
      IF(IND.LT.0.OR.IER.GT.0) GO TO 81
246   CONTINUE
 81   CONTINUE
      IF(ABS(Y(1)).LT.ERR(1)) GO TO 88
      IF(Y(1).GT.0) GO TO 100
      IF(Y(1).LT.0) GO TO 89
100   CONTINUE
      BINT=YO
      YO=(AINT+BINT)/2.
      GO TO 101
 89   CONTINUE
      AINT=YO
      YO=(AINT+BINT)/2.
      GO TO 101
 88   CONTINUE
C-*************************************************************
C-*************************************************************
C-*************************************************************
C-
C-----CALCULATION OF DEFLECTION AND SLOPE AS A FUNCTION
C-      OF X FOR SEGMENT 1
C-
      WRITE(6,500) YO
500   FORMAT(//////////,10X,'YO=',F14.11,)
      WRITE(6,501)
501   FORMAT('----------------------------------------')
      WRITE(6,502)
```

```
502   FORMAT('----------------------------------------')                 00024900
      WRITE(6,2)                                                          00025000
  2   FORMAT(/////,' DEFLECTION AND SLOPE FOR SEGMENT 1',//)              00025100
      WRITE(6,9)                                                          00025200
  9   FORMAT(10X,'X DISTANCE',5X,'DEFLECTION',7X,'SLOPE',9X,'STRESS'      00025300
     -,8X,'MOMENT',//)                                                    00025400
      JW=0                                                                00025500
      DO 505 X=0.0,SEGA+.005,SEGA/STEP                                    00025600
      PROD(3)=SQRT(PLOAD/(ESMC*(WIDTH*THICK**3./12.)))                    00025700
      PROD(1)=YO*(.5*(EXP(PROD(3)*X)-EXP(-PROD(3)*X)))                    00025800
      PROD(2)=.5*(EXP(PROD(3)*SEGA)-EXP(-PROD(3)*SEGA))                   00025900
      T(1)=PROD(1)/PROD(2)                                                00026000
      T(2)=YO*PROD(3)/PROD(2)*(.5*(EXP(PROD(3)*X)+EXP                     00026100
     -(-PROD(3)*X)))                                                      00026200
C-----CALCULATION OF TOP AND BOTTOM FIBER STRESSES                        00026220
      SIGT=-PLOAD*T(1)*THICK/2./(1./12.*WIDTH*THICK**3)+PLOAD/            00026250
     -(WIDTH*THICK)                                                       00026251
      SIGB=PLOAD*T(1)*THICK/2./(1./12.*WIDTH*THICK**3)+PLOAD/             00026252
     -(WIDTH*THICK)                                                       00026253
      J=JW+2                                                              00026300
      IF(INT(J/2).NE.J/2.0) GO TO 199                                     00026400
      WRITE(6,10) X, T(1), T(2), SIGT, PLOAD*T(1)                         00026500
 10   FORMAT(10X,5(E11.4,3X))                                             00026600
      WRITE(6,11) SIGB                                                    00026650
 11   FORMAT(52X,E11.4,//)                                                C0026660
199   JW=JW+1                                                             C0026700
505   CONTINUE                                                            C0026800
C-                                                                        C0026900
C-----CALCULATION OF DEFLECTION AND SLOPE AS A FUNCTION                   C0027000
C-      OF ARC LENGTH FOR SEGMENT 2                                       C0027100
C-                                                                        C0027200
      WRITE(6,24)                                                         C0027300
 24   FORMAT(/////,' DEFLECTION AND SLOPE FOR SEGMENT 2',//)              C0027400
      WRITE(6,25)                                                         C0027500
 25   FORMAT(10X,'ARC LENGTH',5X,'DEFLECTION',7X,'SLOPE',10X,'STRESS'     C0027600
     -,12X,'MOMENT',11X,'SHEAR',//)                                       C0027700
      EXTERNAL FCN1                                                       C0027900
      R=WIDTH*THICK/(ALOG(RADO/RADI))                                     C0028000
      NW=2                                                                C0028100
      THETA=ARCOS((5.*THICK-BOND)/(6.*THICK))                             C0028200
      N=2                                                                 C0028300
      YBAR=RADI+THICK/2.-R                                                C0028400
      X=0.0                                                               C0028500
      Y(1)=-T(1)                                                          C0028600
      Y(2)=-T(2)                                                          C0028700
      TOL=.000001                                                         C0028900
      IND=1                                                               C0029000
      J=0                                                                 C0029100
      DO 510 Z=0.0,R*THETA+.001,R*THETA/STEP                              C0029200
      XEND=FLOAT(Z)                                                       C0029300
      CALL DVERK(N,FCN1,X,Y,XEND,TOL,IND,C,NW,W,IER)                      C0029400
      IF(IND.LT.0.OR.IER.GT.0) GO TO 511                                  C0029500
      JW=J+10                                                             C0029600
      IF(INT(JW/10).NE.JW/10.0) GO TO 299                                 C0029700
      WRITE(6,30) X,Y(1),Y(2), TORC, SHEAR                                C0029800
 30   FORMAT(10X,3(E11.4,3X),20X,E11.4,5X,E11.4,/)                        C0029900
      YGLOB=-.5*THICK                                                     C0029940
      DO 660 H=-(RADO-R),R-RADI,THICK/10.                                 C0029950
C-----STRESSES FROM THE MOMENT DISTRIBUTION SUPERIMPOSED                   C0029955
C-----ON TENSILE STRESSES AS A FUNCTION OF BEAM THICKNESS                  C0029957
      AREA=WIDTH*THICK                                                    C0029959
      BSIGX=TORC*H/((R-H)*WIDTH*THICK*YBAR)+PLOAD*COS(X/R)/AREA            C0029960
```

```
      WRITE(6.600) YGLOB. BSIGX
  600 FORMAT(52X,F4.2,1X,E11.4)
      YGLOB=YGLOB+THICK/10.
 660  CONTINUE
 299  J=J+1
 510  CONTINUE
 511  CONTINUE
C-
C-----CALCULATION OF DEFLECTION AND SLOPE FOR SEGMENT 3
C-     AS A FUNCTION O- ARC LENGTH
C-
      WRITE(6.27)
   27 FORMAT(/////.' DEFLECTION AND SLOPE FOR SEGMENT 3'.//)
      WRITE(6.26)
   26 FORMAT(10X.'ARC LENGTH'.5X.'DEFLECTION',7X.'SLOPE'.10X.'STRESS'
     -.12X.'MOMENT'.11X.'SHEAR'.//)
      EXTERNAL FCN2
      X=0.0
      Y(1)=-Y(1)
      Y(2)=-Y(2)
      NW=2
      N=2
      IND=1
      DO 515 M=1.24
      C(M)=0.0
 515  CONTINUE
      J=0
      DO 520 Z=0.0.R*THETA+.001  .R*THETA/STEP
      XEND=FLOAT(Z)
      CALL DVERK(N.FCN2.X.Y.XEND.TOL.IND.C.NW.W.IER)
      IF(IND.LT.0.OR.IER.GT.0) GO TO 525
      JW=J+10
      IF(INT(JW/10).NE.JW/10.0) GO TO 349
      WRITE(6.60) X. Y(1). Y(2). TORCS. SHEAR
   60 FORMAT(10X.3(E11.4.3X).20X.E11.4.5X.E11.4./)
      YGLOB=-.5*THICK
      DO 770 H=-(RADO-R).R-RADI.THICK/10.
C-----STRESSES FROM THE MOMENT DISTRIBUTION SUPERIMPOSED
C-----ON TENSILE STRESSES AS A FUNCTION OF BEAM THICKNESS
      CSIGX=-TORCS*H/((R-H)*WIDTH*THICK*YBAR)+PLOAD*COS(THETA-X/R)/
     -(WIDTH*THICK)
      WRITE(6.700) YGLOB. CSIGX
 700  FORMAT(52X.F4.2.1X.E11.4)
      YGLOB=YGLOB+THICK/10.
 770  CONTINUE
 349  J=J+1
 520  CONTINUE
 525  CONTINUE
C-
C-----CALCULATION OF DEFLECTION AND SLOPE AS A FUNCTION
C-     OF X FOR SEGMENT 4
C-
      WRITE(6.28)
   28 FORMAT(/////.' DEFLECTION AND SLOPE FOR SEGMENT 4'.//)
      WRITE(6.29)
   29 FORMAT(10X.'X DISTANCE'.5X.'DEFLECTION'.7X.'SLOPE'.//)
      EXTERNAL FCN3
      X=0.0
      NW=2
      N=2
      IND=1
      DO 530 M=1.24
```

```
      C(M)=0.0
 530  CONTINUE
      J=0
      DO 535 Z=0.0,CONTA+.005,CONTA/STEP
      XEND=FLOAT(Z)
      CALL DVERK(N,FCN3,X,Y,XEND,TOL,IND,C,NW,W,IER)
      IF(IND.LT.0.OR.IER.GT.0) GO TO 540
      JW=J+10
      IF(INT(JW/10).NE.JW/10.0) GO TO 399
      WRITE(6,62) X, Y(1), Y(2)
  62  FORMAT(10X,3(E11.4,3X),/)
 399  J=J+1
 535  CONTINUE
 540  CONTINUE
C-
C-----CALCULATION OF DEFLECTION AND SLOPE AS A FUNCTION OF
C-      X FOR SEGMENT 5
C-
      WRITE(6,31)
  31  FORMAT(/////,' DEFLECTION AND SLOPE FOR SEGMENT 5',//)
      WRITE(6,32)
  32  FORMAT(10X,'X DISTANCE',5X,'DEFLECTION',7X,'SLOPE',10X,'STRESS'
     -,12X,'MOMENT',//)
      EXTERNAL FCN4
      X=0.0
      NW=2
      N=2
      IND=1
      DO 545 M=1,24
      C(M)=0.0
 545  CONTINUE
      J=0
      DO 550 Z=0.0,SEGB+.005,SEGB/STEP
      XEND=FLOAT(Z)
      CALL DVERK(N,FCN4,X,Y,XEND,TOL,IND,C,NW,W,IER)
      IF(IND.LT.0.OR.IER.GT.0) GO TO 555
      JW=J+10
      IF(X.EQ..15) GO TO 244
      IF(INT(JW/10).NE.JW/10.0) GO TO 245
 244  WRITE(6,65) X, Y(1), Y(2), PLOAD*Y(1)
  65  FORMAT(10X,3(E11.4,3X),20X,E11.4,/)
      DO 880 H=-THICK/2.,THICK/2.,THICK/10.
C-----STRESSES FROM THE MOMENT DISTRIBUTION SUPERIMPOSED
C-----ON TENSILE STRESSES AS A FUNCTION OF BEAM THICKNESS
      ESIGX=-PLOAD*Y(1)*H/(1./12.*WIDTH*THICK**3)+PLOAD/(WIDTH*THICK)
      WRITE(6,800) H, ESIGX
 800  FORMAT(52X,F4.2,1X,E11.4)
 880  CONTINUE
 245  J=J+1
 550  CONTINUE
 555  CONTINUE
      STOP
      END
C-*************************************************************
C-*************************************************************
C-***                      SUBROUTINES                      ***
C-*************************************************************
C-*************************************************************
C-
C-----NUMERICAL INTEGRATION OF SEG2
C-
      SUBROUTINE FCN1(N,X,Y,YPRIME)
```

```
      COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI
     -, BOND, J, TORC, TORCS, TRAC, TRACE, SHEAR
      DIMENSION Y(2), YPRIME(2)
      YPRIME(1)=Y(2)
C-
C-----ECCENTRICITY DUE ONLY TO EXTENSIONAL EFFECTS
C-
      YECC=PLOAD*X*COS(X/R)*SIN(X/R)/(WIDTH*THICK*ESMC)
      TORC=PLOAD*(YECC*TRAC+YBAR+R*(1.-COS(X/R))-Y(1)*COS(X/R))
      SHEAR=PLOAD*(PLOAD/(WIDTH*THICK*ESMC)*(X/R*COS(X/R)**2
     -+SIN(X/R)*(X/R*(-SIN(X/R))+COS(X/R)))+SIN(X/R)+Y(1)/R
     -*SIN(X/R))
 560  YPRIME(2)=(R+Y(1))*TORC/(R**2.*ESMC*WIDTH*THICK*YBAR)
      RETURN
      END
C-
C-----NUMERICAL INTEGRATION OF SEG3
C-
      SUBROUTINE FCN2(N,X,Y,YPRIME)
      COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI
     -, BOND, J, TORC, TORCS, TRAC, TRACE, SHEAR
      DIMENSION Y(2), YPRIME(2)
      YPRIME(1)=Y(2)
C-
C-----ECCENTRICITY DUE TO GEOMETRY
C-
      AEGEO=YBAR+R*(1.-COS(THETA))-2.*YBAR*COS(THETA)
C-
C-----ECCENTRICITY DUE TO GEOMETRY
C-
      BEGEO=R*(SIN(X/R+PI/2.-THETA)-SIN(PI/2.-THETA))
C-
C-----ECCENTRICITY DUE ONLY TO EXTENSIONAL EFFECTS
C-
      EEXT=PLOAD*(SIN(PI/2.-THETA+X/R))*SIN(THETA-X/R)*X/(
     -WIDTH*THICK*ESMC)
C-
C-----ECCENTRICITY DUE TO DEFLECTION
C-
      EDEFL=Y(1)*COS(THETA-X/R)
      TORCS=PLOAD*(AEGEO+BEGEO+EDEFL+EEXT*TRACE)
      SHEAR=PLOAD*(COS(X/R+PI/2.-THETA)-Y(1)/R*SIN(PI/2.
     --THETA+X/R)+PLOAD*((SIN(PI/2.-THETA+X/R))*(SIN(THETA-
     -X/R)/(WIDTH*THICK*ESMC)-X/(WIDTH*THICK*R*ESMC)*COS
     -(THETA-X/R))+X/(WIDTH*THICK*ESMC*R)*SIN(THETA-X/R)
     -*COS(PI/2.-THETA+X/R)))
      YPRIME(2)=(R+Y(1))*TORCS/(R**2.*ESMC*WIDTH*THICK*YBAR)
      RETURN
      END
C-
C-----NUMERICAL INTEGRATION OF SEG4
C-
      SUBROUTINE FCN3(N,X,Y,YPRIME)
      COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI
     -, BOND
      DIMENSION Y(2), YPRIME(2), EI(3)
      EI(1)=ESMC*(1./12.*THICK**3.*WIDTH+WIDTH*THICK*(.5*THICK+BOND
     -/2.)**2.)
      EI(2)=EADH*1./12.*THICK**3*WIDTH
      EI(3)=ESMC*(1./12.*WIDTH*THICK**3+WIDTH*THICK*(.5*THICK+BOND/
     -2.)**2.)
      YPRIME(1)=Y(2)
```

```
      DEN=EI(1)+EI(2)+EI(3)
      YPRIME(2)=PLOAD*(.5*THICK+BOND/2.+Y(1))/DEN
      RETURN
      END
C-
C-----NUMERICAL INTEGRATION OF SEG5
C-
      SUBROUTINE FCN4(N,X,Y,YPRIME)
      COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI
     -, BOND
      DIMENSION Y(2), YPRIME(2)
      YPRIME(1)=Y(2)
      YPRIME(2)=PLOAD*(Y(1))/(ESMC*WIDTH*THICK**3./12.)
      RETURN
      END
```

b. CONVERT

The program CONVERT essentially performs the tedious calculations involved in computing the boundary conditions for the finite-element model. Stresses dictated by the beam bending model are converted to equivalent point forces which are then applied to the finely meshed ends of the finite-element structure. In converting the stress distribution from deformed to undeformed geometry the program insures that the model be maintained in equilibrium through the introduction of a correcting moment.

The important parameters utilized in the routine are defined in the nomenclature section of the program. Frequent comment cards are intended to assist the user in the utilization of the program.

```
C-     CCCCC   OOOOO   N   N   V   V   EEEEE   RRRRR   TTTTT
C-     C       O   O   NN  N   V   V   E       R   R     T
C-     C       O   O   N N N   V   V   EEE     RRRRR     T
C-     C       O   O   N  NN    V V    E       R R       T
C-     CCCCC   OOOOO   N   N     V     EEEEE   R  R      T
C-
C-
C-
C-
C-
C-
C-                  STRESS TRANSFORMATION PROGRAM
C-
C-                    DEVELOPED BY:  RICHARD C. GIVLER
C-                        UNIVERSITY OF DELAWARE
C-                          OCT 78 - JAN 79
C-
C-
C-
C-
C-
C-
C-
C-
$RESET FREE
      DIMENSION ZOM(5), RES(5), PART(4), ZOMX(5), RESB(5)
     -, RESX(5), RESY(5), ZOMA(5), ZOMB(5), BMOM(5), ZOMXB(5)
     -, RESXB(5), RESYB(5)
C-
C-----PARAMETERS AND NOMENCLATURE
C-
C------MATERIAL THICKNESS
      THICK=.1
C------MATERIAL WIDTH
      WIDTH=1.
C------ROTATION OF LEFT HAND FACE FROM UNDEFORMED GEOMETRY (RAD)
      DUDSA=-.01642
C------ROTATION OF RIGHT HAND FACE FROM UNDEFORMED GEOMETRY (RAD)
      DUDSB=.008420
C------TOTAL MOMENT ON LEFT HAND FACE (IN. LBS.)
      TOTMOA=10.64
C------SHEAR ON LEFT HAND FACE
      SHEARA=132.6
C------TOTAL MOMENT ON RIGHT HAND FACE (IN. LBS.)
      TOTMOB=-1.968
      AREA=WIDTH*THICK
C------OUTSIDE RADIUS IN INCHES
      RADO=3.5*THICK
C------INSIDE RADIUS IN INCHES
      RADI=2.5*THICK
C------RADIUS OF CURVATURE OF CURVED MEMBERS
      R=WIDTH*THICK/(ALOG(RADO/RADI))
C------DIFFERENCE BETWEEN NEUTRAL AXIS AND CENTROIDAL AXIS (IN.)
      YBAR=RADI+THICK/2.-R
C------ADHESIVE BOND THICKNESS IN INCHES
      BOND=.03
C------ANGLE SUBTENDED BY CURVED MEMBERS IN RADIANS
      THETA=ARCOS((5.*THICK-BOND)/(6.*THICK))
C------TENSILE LOAD
      PLOAD=200.
C-
```

```
C------NOTE:  DEFLECTIONS ARE MEASURED NORMAL TO THE NEUTRAL AXIS
C-
C------DEFLECTION OF LEFT HAND FACE (IN.)
      DEFLA=-.01547
C------DEFLECTION OF RIGHT HAND FACE (IN.)
      DEFLB=-.007872
C------
C-*******************************************************
C------
C------RESOLVING STRESS DISTRIBUTION ON LEFT HAND FACE
C------
C-*******************************************************
C------
C------
C------CALCULATION OF RESULTANT POINT FORCES FROM THE
C------STRESS DISTRIBUTION
C------
      H=R-RADI
      DO 100 N=1,5
      PART(1)=TOTMOA/(YBAR*AREA)*(R-H-R*ALOG(R-H))
     -+PLOAD/AREA*COS(THETA+DUDSA)*H
      H=H-.02
      PART(2)=TOTMOA/(YBAR*AREA)*(R-H-R*ALOG(R-H))
     -+PLOAD/AREA*COS(THETA+DUDSA)*H
      RES(N)=PART(1)-PART(2)
  100 CONTINUE
C------
C------CALCULATION OF ACTUAL MOMENTS FROM THE STRESS DISTRIBUTION
C------
      H=R-RADI
      DO 200 N=1,5
      PART(3)=TOTMOA/(YBAR*AREA)*(-1.)*(.5*(R-H)**2-2.*R*(R-H)+R**2
     -*ALOG(R-H))+.5*PLOAD/AREA*COS(THETA+DUDSA)*H**2
      H=H-.02
      PART(4)=TOTMOA/(YBAR*AREA)*(-1.)*(.5*(R-H)**2-2.*R*(R-H)+R**2
     -*ALOG(R-H))+.5*PLOAD/AREA*COS(THETA+DUDSA)*H**2
      ZOM(N)=PART(3)-PART(4)
  200 CONTINUE
C------
C------CALCULATION OF CORRECTION MOMENT DUE TO THE
C------REPRESENTATION OF THE STRESS DISTRIBUTION BY POINT
C------FORCES
C------
      H=R-RADI-.01
      DO 300 N=1,5
      ZOMX(N)=ZOM(N)-RES(N)*H
      H=H-.02
  300 CONTINUE
C------
C------CALCULATION OF MOMENT DUE TO TRANSLATION OF THE STRESS
C------DISTRIBUTION THROUGH SPACE FROM THE DEFORMED GEOMETRY
C------TO THE UNDEFORMED GEOMETRY
C------
C------
C------CALCULATION OF MOMX FOR INPUT INTO THE FINITE ELEMENT
C------MODEL
C------
      H=R-RADI-.01
      DO 400 N=1,5
      ZOMA(N)=-RES(N)*((H+DEFLA)*COS(DUDSA)-H)
      ZOMX(N)=ZOMX(N)+ZOMA(N)
      H=H-.02
```

```
  400 CONTINUE
C------
C------CALCULATION OF FY AND FZ FOR INPUT INTO THE FINITE
C------ELEMENT MODEL
C------
      WRITE(6,25)
   25 FORMAT(///////,6X,'BOUNDARY CONDITIONS FOR  LEFT HAND SEGMENT',/)
      WRITE(6,30)
   30 FORMAT(/,4X,'NODE',6X,'FY',13X,'FZ',12X,'MOMX',//)
      DO 500 N=1,5
      RESX(N)=-RES(N)*COS(THETA+DUDSA)-SHEARA/5.*SIN(THETA)
      RESY(N)=-RES(N)*SIN(THETA+DUDSA)+SHEARA/5.*COS(THETA)
      WRITE(6,2) N, RESX(N), RESY(N), -ZOMX(N)
    2 FORMAT(5X,F2.0,3X,3(E10.4,5X),/)
  500 CONTINUE
C------
C------
C-*********************************************************
C------
C------RESOLVING STRESS DISTRIBUTION ON RIGHT HAND FACE
C------
C-*********************************************************
C------
C------
C------
C------CALCULATION OF RESULTANT POINT FORCES FROM THE STRESS
C------DISTRIBUTION
C------
      H=THICK/2.
      DO 1000 N=1,5
      PART(1)=-.5*TOTMOB*H**2/(1./12.*WIDTH*THICK**3)+PLOAD/AREA*H
      H=H-.02
      PART(2)=-.5*TOTMOB*H**2/(1./12.*WIDTH*THICK**3)+PLOAD/AREA*H
      RESB(N)=PART(1)-PART(2)
 1000 CONTINUE
C------
C------CALCULATION OF ACTUAL MOMENTS FROM THE STRESS
C------DISTRIBUTION
C------
      H=THICK/2.
      DO 1100 N=1,5
      PART(3)=1./3.*(TOTMOB)*H**3/(1./12.*WIDTH*THICK**3)
     --.5*PLOAD/AREA*H**2
      H=H-.02
      PART(4)=1./3.*(TOTMOB)*H**3/(1./12.*WIDTH*THICK**3)
     --.5*PLOAD/AREA*H**2
      BMOM(N)=PART(3)-PART(4)
 1100 CONTINUE
C------
C------CALCULATION OF CORRECTION MOMENT DUE TO THE
C------REPRESENTATION OF THE STRESS DISTRIBUTION BY POINT
C------FORCES
C------
      H=THICK/2.-.01
      DO 1200 N=1,5
      ZOMXB(N)=BMOM(N)+RESB(N)*H
      H=H-.02
 1200 CONTINUE
C------
C------CALCULATION OF THE MOMENT DUE TO TRANSLATING THE STRESS
C------DISTRIBUTION FROM DEFORMED TO UNDEFORMED GEOMETRY
C------
```

```
C------
C------CALCULATION OF MOMENT MX FOR INPUT INTO THE FINITE
C------ELEMENT MODEL
C------
      H=THICK/2.-.01
      DO 1300 N=1,5
      ZOMB(N)=RESB(N)*((H-DEFLB)*COS(DUDSB)-H)
      ZOMXB(N)=ZOMXB(N)+ZOMB(N)
      H=H-.02
 1300 CONTINUE
C------
C------CALCULATION OF FY AND FZ FOR INPUT INTO THE FINITE
C------ELEMENT MODEL
C------
      WRITE(6,40)
   40 FORMAT(//////,10X,'BONDARY CONDITIONS FOR SEGB',/)
      WRITE(6,45)
   45 FORMAT(/,4X,'NODE',6X,'FY',13X,'FZ',12X,'MOMX',//)
      DO 1400 N=1,5
      RESXB(N)=RESB(N)*COS(DUDSB)
      RESYB(N)=RESB(N)*SIN(DUDSB)
      WRITE(6,20) N, RESXB(N), RESYB(N), ZOMXB(N)
   20 FORMAT(5X,F2.0,3X,3(E10.4,5X),/)
 1400 CONTINUE
      END
```

```
      SUBROUTINE DVERK  (N,FCN,X,Y,XEND,TOL,IND,C,NW,W,IER)
C
C-DVERK---------S-------LIBRARY 3--------------------------------------
C
C   FUNCTION            - SOLUTION OF A SYSTEM OF FIRST ORDER ORDINARY
C                           DIFFERENTIAL EQUATIONS OF THE FORM
C                           DY/DX = F(X,Y) WITH INITIAL CONDITIONS.
C                           A RUNGE-KUTTA METHOD BASED ON VERNERS FIFTH
C                           AND SIXTH ORDER PAIR OF FORMULAS IS USED.
C   USAGE               - CALL DVERK(N,FCN,X,Y,XEND,TOL,IND,C,NW,W,IER)
C   PARAMETERS   N      - NUMBER OF EQUATIONS. (INPUT)
C                FCN    - NAME OF SUBROUTINE FOR EVALUATING FUNCTIONS.
C                           (INPUT)
C                           THE SUBROUTINE ITSELF MUST ALSO BE PROVIDED
C                             BY THE USER AND IT SHOULD BE OF THE
C                             FOLLOWING FORM
C                               SUBROUTINE FCN(N,X,Y,YPRIME)
C                               DIMENSION Y(N),YPRIME(N)
C                                     .
C                                     .
C                                     .
C                           FCN SHOULD EVALUATE YPRIME(1),....,YPRIME(N)
C                             GIVEN N,X, AND Y(1),....,Y(N).  YPRIME(I)
C                             IS THE FIRST DERIVATIVE OF Y(I) WITH
C                             RESPECT TO X.
C                           FCN MUST APPEAR IN AN EXTERNAL STATEMENT IN
C                             THE CALLING PROGRAM AND N,X,Y(1),....Y(N)
C                             MUST NOT BE ALTERED BY FCN.
C                X      - INDEPENDENT VARIABLE. (INPUT AND OUTPUT)
C                           ON INPUT, X SUPPLIES THE INITIAL VALUE.
C                           ON OUTPUT, X IS REPLACED WITH XEND UNLESS
C                             ERROR CONDITIONS ARISE.  SEE THE DES-
C                             CRIPTION OF PARAMETER IND.
C                Y      - DEPENDENT VARIABLES, VECTOR OF LENGTH N.
C                           (INPUT AND OUTPUT)
C                           ON INPUT, Y(1),....,Y(N) SUPPLY INITIAL
C                             VALUES.
C                           ON OUTPUT, Y(1),....,Y(N) ARE REPLACED WITH
C                             AN APPROXIMATE SOLUTION AT XEND UNLESS
C                             ERROR CONDITIONS ARISE.  SEE THE DES-
C                             CRIPTION OF PARAMETER IND.
C                XEND   - VALUE OF X AT WHICH SOLUTION IS DESIRED.
C                           (INPUT)
C                           XEND MAY BE LESS THAN THE INITIAL VALUE OF
C                             X.
C                TOL    - TOLERANCE FOR ERROR CONTROL. (INPUT)
C                           THE SUBROUTINE ATTEMPTS TO CONTROL A NORM
C                             OF THE LOCAL ERROR IN SUCH A WAY THAT THE
C                             GLOBAL ERROR IS PROPORTIONAL TO TOL.
C                             MAKING TOL SMALLER IMPROVES ACCURACY AND
C                             MORE THAN ONE RUN, WITH DIFFERENT VALUES
C                             OF TOL, CAN BE USED IN AN ATTEMPT TO
C                             ESTIMATE THE GLOBAL ERROR.
C                           IN THE DEFAULT CASE (IND=1), THE GLOBAL
C                             ERROR IS
C                                 MAX(ABS(E(1)),....,ABS(E(N)))
C                             WHERE E(K)=(Y(K)-YT(K))/MAX(1,ABS(Y(K)))
C                             YT(K) IS THE TRUE SOLUTION, AND
C                             Y(K) IS THE COMPUTED SOLUTION AT XEND,
C                             FOR K=1,2,....,N.
C                             OTHER ERROR CONTROL OPTIONS ARE AVAILABLE.
```

```
C                    SEE THE DESCRIPTION OF PARAMETERS IND AND
C                    C BELOW.
C             IND   - INDICATOR. (INPUT AND OUTPUT)
C                  ON INITIAL ENTRY IND MUST BE SET EQUAL TO
C                  EITHER 1 OR 2.
C                  IND = 1 CAUSES ALL DEFAULT OPTIONS TO BE
C                    USED AND ELIMINATES THE NEED TO SET
C                    SPECIFIC VALUES IN THE COMMUNICATIONS
C                    VECTOR C.
C                  IND = 2 ALLOWS OPTIONS TO BE SELECTED.  IN
C                    THIS CASE, THE FIRST 9 COMPONENTS OF C
C                    MUST BE INITIALIZED TO SELECT OPTIONS AS
C                    DESCRIBED BELOW.
C                THE SUBROUTINE WILL NORMALLY RETURN WITH
C                  IND = 3, HAVING REPLACED THE INITIAL VALUES
C                  OF X AND Y WITH, RESPECTIVELY, THE VALUE
C                  XEND AND AN APPROXIMATION TO Y AT XEND.
C                THE SUBROUTINE CAN BE CALLED REPEATEDLY WITH
C                  NEW VALUES OF XEND WITHOUT CHANGING ANY
C                  OF THE OTHER PARAMETERS.
C                THREE ERROR RETURNS ARE ALSO POSSIBLE, IN
C                  WHICH CASE X AND Y WILL BE THE MOST
C                  RECENTLY ACCEPTED VALUES.
C                  IND = -3 INDICATES THAT THE SUBROUTINE WAS
C                    UNABLE TO SATISFY THE ERROR REQUIREMENT.
C                    THIS MAY MEAN THAT TOL IS TOO SMALL.
C                  IND = -2 INDICATES THAT THE VALUE OF HMIN
C                    (MINIMUM STEP-SIZE) IS GREATER THAN HMAX
C                    (MAXIMUM STEP-SIZE), WHICH PROBABLY MEANS
C                    THAT THE REQUESTED TOL (WHICH IS USED IN
C                    THE CALCULATION OF HMIN) IS TOO SMALL.
C                  IND = -1 INDICATES THAT THE ALLOWED MAXIMUM
C                    NUMBER OF FCN EVALUATIONS HAS BEEN
C                    EXCEEDED.  THIS CAN ONLY OCCUR IF OPTION
C                    C(7), AS DESCRIBED BELOW, HAS BEEN USED.
C             C     - COMMUNICATIONS VECTOR OF LENGTH 24. (INPUT IF
C                  IND.NE.1, AND OUTPUT).
C                  C IS USED TO SELECT OPTIONS AND TO RETAIN
C                    INFORMATION BETWEEN CALLS.  THE USER NEED
C                    NOT BE CONCERNED WITH THE FOLLOWING
C                    DESCRIPTION OF THE ELEMENTS OF C WHEN
C                    DEFAULT OPTIONS ARE USED (IND=1).
C                    HOWEVER, WHEN IT IS DESIRED TO USE IND=2
C                    AND SELECT OPTIONS, A BASIC UNDERSTANDING
C                    OF DVERK IS REQUIRED.  THE FOLLOWING
C                    PARAGRAPH DESCRIBES, BRIEFLY, THE BASIC
C                    TERMS.  FOR MORE DETAILS, SEE THE
C                    REFERENCE.
C                    DVERK ADVANCES THE INDEPENDENT VARIABLE
C                    X ONE STEP AT A TIME UNTIL XEND IS
C                    REACHED.  THE SOLUTION IS COMPUTED AT
C                    XTRIAL = X+HTRIAL ALONG WITH AN ERROR
C                    ESTIMATE EST.  IF EST IS LESS THAN OR
C                    EQUAL TO TOL (SUCCESSFUL STEP), THE STEP
C                    IS ACCEPTED AND X IS ADVANCED TO XTRIAL.
C                    IF EST IS GREATER THAN TOL (FAILURE)
C                    HTRIAL IS ADJUSTED AND THE SOLUTION IS
C                    RECOMPUTED.  HMAG = ABS(HTRIAL) IS NEVER
C                    ALLOWED TO EXCEED HMAX NOR IS IT ALLOWED
C                    TO BECOME SMALLER THAN HMIN.  THE FIRST
C                    TRIAL STEP IS HSTART.  DURING THE
C                    COMPUTATION, A COUNTER (C(23)) IS
```

```
C              INCREMENTED EACH TIME A TRIAL STEP FAILS  DVEK1240
C              TO PROVIDE A SOLUTION SATISFYING THE ERRORDVEK1250
C              TOLERANCE.  ANOTHER COUNTER (C(22)) IS    DVEK1260
C              USED TO RECORD THE NUMBER OF SUCCESSFUL   DVEK1270
C              STEPS.  AFTER A SUCCESSFUL STEP, C(23) IS DVEK1280
C              SET TO ZERO.                              DVEK1290
C          OPTIONS.  IF THE SUBROUTINE IS ENTERED WITH   DVEK1300
C            IND=2, THE FIRST 9 COMPONENTS OF THE        DVEK1310
C            COMMUNICATIONS VECTOR MUST BE INITIALIZED   DVEK1320
C            BY THE USER.  NORMALLY THIS IS DONE BY      DVEK1330
C            FIRST SETTING THEM ALL TO ZERO, AND THEN    DVEK1340
C            THOSE CORRESPONDING TO PARTICULAR OPTIONS   DVEK1350
C            ARE MADE NON-ZERO.                          DVEK1360
C   C(1)   - ERROR CONTROL INDICATOR.                    DVEK1370
C            THE SUBROUTINE ATTEMPTS TO CONTROL A NORM   DVEK1380
C            OF THE LOCAL ERROR IN SUCH A WAY THAT THE   DVEK1390
C            GLOBAL ERROR IS PROPORTIONAL TO TOL.        DVEK1400
C            THE DEFINITION OF GLOBAL ERROR FOR THE      DVEK1410
C            DEFAULT CASE (IND=1) IS GIVEN IN THE        DVEK1420
C            DESCRIPTION OF PARAMETER TOL.  THE DEFAULT  DVEK1430
C            WEIGHTS ARE 1/MAX(1,ABS(Y(K))).  WHEN IND=2 DVEK1440
C            IS USED, THE WEIGHTS ARE DETERMINED         DVEK1450
C            ACCORDING TO THE VALUE OF C(1).             DVEK1460
C            IF C(1)=1 THE WEIGHTS ARE 1                 DVEK1470
C                       (ABSOLUTE ERROR CONTROL)         DVEK1480
C            IF C(1)=2 THE WEIGHTS ARE 1/ABS(Y(K))       DVEK1490
C                      FOR K=1,2,...,N.                  DVEK1500
C                      (RELATIVE ERROR CONTROL)          DVEK1510
C            IF C(1)=3 THE WEIGHTS ARE                   DVEK1520
C                      1/MAX(ABS(C(2)),ABS(Y(K)))        DVEK1530
C                      FOR K=1,2,...,N.                  DVEK1540
C                      (RELATIVE ERROR CONTROL, UNLESS   DVEK1550
C                      ABS(Y(K)) IS LESS THAN THE FLOOR  DVEK1560
C                      VALUE,ABS(C(2)))                  DVEK1570
C            IF C(1)=4 THE WEIGHTS ARE                   DVEK1580
C                      1/MAX(ABS(C(K+30)),ABS(Y(K)))     DVEK1590
C                      FOR K=1,2,...,N.                  DVEK1600
C                      (HERE INDIVIDUAL FLOOR VALUES     DVEK1610
C                      ARE USED)                         DVEK1620
C                      IN THIS CASE,  THE DIMENSION OF C DVEK1630
C                      MUST BE GREATER THAN OR EQUAL TO  DVEK1640
C                      N+30 AND C(31), C(32),...,C(N+30) DVEK1650
C                      MUST BE INITIALIZED BY THE USER.  DVEK1660
C            IF C(1)=5 THE WEIGHTS ARE 1/ABS(C(K+30))    DVEK1670
C                      FOR K=1,2,...,N.                  DVEK1680
C                      IN THIS CASE,  THE DIMENSION OF C DVEK1690
C                      MUST BE GREATER THAN OR EQUAL TO  DVEK1700
C                      N+30 AND C(31), C(32),...,C(N+30) DVEK1710
C                      MUST BE INITIALIZED BY THE USER.  DVEK1720
C            FOR ALL OTHER VALUES OF C(1), INCLUDING     DVEK1730
C               C(1)=0 THE DEFAULT VALUES OF             DVEK1740
C                      THE WEIGHTS ARE TAKEN TO BE       DVEK1750
C                      1/MAX(1,ABS(Y(K)))                DVEK1760
C                      FOR K=1,2,...,N.                  DVEK1770
C   C(2)   - FLOOR VALUE.  USED WHEN THE INDICATOR C(1)  DVEK1780
C            HAS THE VALUE 3.                            DVEK1790
C   C(3)   - HMIN SPECIFICATION.  IF NOT ZERO, THE SUB-  DVEK1800
C            ROUTINE CHOOSES HMIN TO BE ABS(C(3)).       DVEK1810
C            OTHERWISE IT USES THE DEFAULT VALUE         DVEK1820
C            10*MAX(DWARF,RREB*MAX(NORM(Y)/TOL,ABS(X)))  DVEK1830
C            WHERE DWARF IS A VERY SMALL POSITIVE MACHINEDVEK1840
C            NUMBER AND RREB IS THE RELATIVE ROUNDOFF    DVEK1850
```

```
C                    ERROR BOUND.
C         C(4)    - HSTART SPECIFICATION.  IF NOT ZERO, THE SUB-
C                    ROUTINE WILL USE AN INITIAL HMAG EQUAL TO
C                    ABS(C(4)), EXCEPT OF COURSE FOR THE RE-
C                    STRICTIONS IMPOSED BY HMIN AND HMAX.
C                    OTHERWISE IT USES THE DEFAULT VALUE
C                      HMAX*(TOL)**(1/6).
C         C(5)    - SCALE SPECIFICATION.  THIS IS INTENDED TO BE
C                    A MEASURE OF THE SCALE OF THE PROBLEM.
C                    LARGER VALUES OF SCALE TEND TO MAKE THE
C                    METHOD MORE RELIABLE, FIRST BY POSSIBLY RE-
C                    STRICTING HMAX (AS DESCRIBED BELOW) AND
C                    SECOND, BY TIGHTENING THE ACCEPTANCE
C                    REQUIREMENT.  IF C(5) IS ZERO, A DEFAULT
C                    VALUE OF 1 IS USED. FOR LINEAR HOMOGENEOUS
C                    PROBLEMS WITH CONSTANT COEFFICIENTS, AN
C                    APPROPRIATE VALUE FOR SCALE IS A NORM OF
C                    THE ASSOCIATED MATRIX. FOR OTHER PROBLEMS,
C                    AN APPROXIMATION TO AN AVERAGE VALUE OF A
C                    NORM OF THE JACOBIAN ALONG THE TRAJEC-
C                    TORY MAY BE APPROPRIATE.
C         C(6)    - HMAX SPECIFICATION.  FOUR CASES ARE POSSIBLE.
C                    IF C(6).NE.0 AND C(5).NE.0, HMAX IS TAKEN
C                      TO BE MIN(ABS(C(6)),2/ABS(C(5))).
C                    IF C(6).NE.0 AND C(5).EQ.0, HMAX IS TAKEN
C                      TO BE ABS(C(6)).
C                    IF C(6).EQ.0 AND C(5).NE.0, HMAX IS TAKEN
C                      TO BE 2/ABS(C(5)).
C                    IF C(6).EQ.0 AND C(5).EQ.0, HMAX IS GIVEN
C                      A DEFAULT VALUE OF 2.
C         C(7)    - MAXIMUM NUMBER OF FUNCTION EVALUATIONS.  IF
C                    NOT ZERO, AN ERROR RETURN WITH IND = -1
C                    WILL BE CAUSED WHEN THE NUMBER OF FUNCTION
C                    EVALUATIONS EXCEEDS ABS(C(7)).
C         C(8)    - INTERRUPT NUMBER 1 . IF NOT ZERO, THE SUB-
C                    ROUTINE WILL INTERRUPT THE CALCULATIONS
C                    AFTER IT HAS CHOSEN ITS PRELIMINARY VALUE
C                    OF HMAG, AND JUST BEFORE CHOOSING HTRIAL
C                    AND XTRIAL IN PREPARATION FOR TAKING A STEP
C                    (HTRIAL MAY DIFFER FROM HMAG IN SIGN, AND
C                    MAY REQUIRE ADJUSTMENT IF XEND IS NEAR).
C                    THE SUBROUTINE RETURNS WITH IND = 4, AND
C                    WILL RESUME CALCULATION AT THE POINT OF
C                    INTERRUPTION IF  RE-ENTERED WITH IND = 4.
C         C(9)    - INTERRUPT NUMBER 2.  IF NOT ZERO, THE SUB-
C                    ROUTINE WILL INTERRUPT THE CALCULATIONS
C                    IMMEDIATELY AFTER IT HAS DECIDED WHETHER OR
C                    NOT TO ACCEPT THE RESULT OF THE MOST RECENT
C                    TRIAL STEP, WITH IND = 5 IF IT PLANS TO
C                    ACCEPT, OR IND = 6 IF IT PLANS TO REJECT.
C                    Y(*) IS THE PREVIOUSLY ACCEPTED RESULT,
C                    WHILE W(*,9) IS THE NEWLY COMPUTED TRIAL
C                    VALUE, AND W(*,2) IS THE UNWEIGHTED ERROR
C                    ESTIMATE VECTOR. THE SUBROUTINE WILL RESUME
C                    CALCULATIONS AT THE POINT OF INTERRUPTION
C                    ON RE-ENTRY WITH IND = 5 OR 6.
C                    IND MAY BE CHANGED BY THE USER IN ORDER TO
C                    FORCE ACCEPTANCE OF A STEP (BY CHANGING IND
C                    FROM 6 TO 5) THAT WOULD OTHERWISE BE
C                    REJECTED, OR VICE VERSA.
C         NW      - THE FIRST DIMENSION OF W AS IT APPEARS IN THE
C                    CALLING PROGRAM. (INPUT)
```

```
C                                  NW MUST BE GREATER THAN OR EQUAL TO N.
C                W               - WORKSPACE MATRIX.
C                                    THE FIRST DIMENSION OF W MUST BE NW AND THE
C                                    SECOND MUST BE GREATER THAN OR EQUAL TO 9.
C                IER             - ERROR PARAMETER. (OUTPUT)
C                                  TERMINAL ERRORS
C                                    IER = 129, NW IS LESS THAN N OR TOL IS LESS
C                                               THAN OR EQUAL TO ZERO.
C                                    IER = 130, IND IS NOT IN THE RANGE 1 TO 6.
C                                    IER = 131, XEND HAS NOT BEEN CHANGED FROM
C                                               PREVIOUS CALL OR X IS NOT SET TO
C                                               THE PREVIOUS XEND VALUE.
C                                    IER = 132, THE RELATIVE ERROR CONTROL
C                                               OPTION (C(1)=2) WAS SELECTED AND
C                                               ONE OF THE SOLUTION COMPONENTS
C                                               IS ZERO.
C   PRECISION            - SINGLE
C   REQD. IMSL ROUTINES - UERTST
C   LANGUAGE             - FORTRAN
C-----------------------------------------------------------------------
C   LATEST REVISION      - DECEMBER 15, 1976
C                                    BGH
      INTEGER            N,IND,NW,K
      INTEGER            IER
      REAL               X,Y(N),XEND,TOL,C(1),W(NW,9),TEMP
      REAL               ZERO,ONE,TWO,THREE,FOUR,FIVE,SEVEN,TEN,HALF,P9,
     1                   C4D15,C2D3,C5D6,C1D6,C1D15,C2D96
      REAL               RK(39),REPS,RTOL
      DATA               ZERO/0.0/,ONE/1.0/,TWO/2.0/,THREE/3.0/
      DATA               FOUR/4.0/,FIVE/5.0/,SEVEN/7.0/
      DATA               TEN/10.0/,HALF/0.5/,P9/0.9/
      DATA               C4D15/.26666666667/
      DATA               C2D3/.66666666667/
      DATA               C5D6/.83333333333/
      DATA               C1D6/.16666666667/
      DATA               C1D15/.66666666667E-1/
      DATA               C2D96/120.42729108/
      DATA               REPS/01301000000000000/
      DATA               RTOL/01631000000000000/
      DATA               RK( 1)/.16666666667E+00/
      DATA               RK( 2)/.53333333333E-01/
      DATA               RK( 3)/.21333333333E+00/
      DATA               RK( 4)/.83333333333E+00/
      DATA               RK( 5)/.26666666667E+01/
      DATA               RK( 6)/.25000000000E+01/
      DATA               RK( 7)/.25781250000E+01/
      DATA               RK( 8)/.91666666667E+01/
      DATA               RK( 9)/.66406250000E+01/
      DATA               RK(10)/.88541666667E+00/
      DATA               RK(11)/.24000000000E+01/
      DATA               RK(12)/.80000000000E+01/
      DATA               RK(13)/.65604575163E+01/
      DATA               RK(14)/.30555555556E+00/
      DATA               RK(15)/.34509803922E+00/
      DATA               RK(16)/.55086666667E+00/
      DATA               RK(17)/.16533333333E+01/
      DATA               RK(18)/.94558823529E+00/
      DATA               RK(19)/.32400000000E+00/
      DATA               RK(20)/.23378823529E+00/
      DATA               RK(21)/.20354651163E+01/
      DATA               RK(22)/.69767441860E+01/
      DATA               RK(23)/.56481798146E+01/
```

```
      DATA            RK(24)/.13738156761E+00/
      DATA            RK(25)/.28630226610E+00/
      DATA            RK(26)/.14417855672E+00/
      DATA            RK(27)/.75000000000E-01/
      DATA            RK(28)/.38992869875E+00/
      DATA            RK(29)/.31944444444E+00/
      DATA            RK(30)/.13503836317E+00/
      DATA            RK(31)/.10783298827E-01/
      DATA            RK(32)/.69805194805E-01/
      DATA            RK(33)/.62500000000E-02/
      DATA            RK(34)/.69630124777E-02/
      DATA            RK(35)/.69444144444E-02/
      DATA            RK(36)/.61381074169E-02/
      DATA            RK(37)/.68181818182E-01/
      DATA            RK(38)/.10783298827E-01/
      DATA            RK(39)/.69805194805E-01/
C
C                                  BEGIN INITIALIZATION, PARAMETER
C                                    CHECKING, INTERRUPT RE-ENTRIES
      IER = 0
C                                  ABORT IF IND OUT OF RANGE 1 TO 6
      IF (IND.LT.1.OR.IND.GT.6) GO TO 290
C                                  CASES - INITIAL ENTRY, NORMAL
C                                    RE-ENTRY, INTERRUPT RE-ENTRIES
      GO TO (5,5,40,145,265,265), IND
C                                  CASE 1 - INITIAL ENTRY (IND .EQ. 1
C                                    OR 2) ABORT IF N.GT.NW OR TOL.LE.0
C
    5 IF (N.GT.NW.OR.TOL.LE.ZERO) GO TO 295
      IF (IND.EQ.2) GO TO 15
C                                  INITIAL ENTRY WITHOUT OPTIONS (IND
C                                    .EQ. 1) SET C(1) TO C(9) EQUAL TO
C                                    0
      DO 10 K=1,9
         C(K) = ZERO
   10 CONTINUE
      GO TO 30
C                                  SUMMARY OF THE COMPONENTS OF THE
C                                    COMMUNICATIONS VECTOR
C                                    PRESCRIBED AT THE OPTION
C                                      OF THE USER
C
C                                    C(1) ERROR CONTROL INDICATOR
C                                    C(2) FLOOR VALUE
C                                    C(3) HMIN SPECIFICATION
C                                    C(4) HSTART SPECIFICATION
C                                    C(5) SCALE SPECIFICATION
C                                    C(6) HMAX SPECIFICATION
C                                    C(7) MAX NO OF FCN EVALS
C                                    C(8) INTERRUPT NO 1
C                                    C(9) INTERRUPT NO 2
C
C                                    DETERMINED BY THE PROGRAM
C
C                                    C(10) RREB(REL ROUNDOFF ERROR BND)
C                                    C(11) DWARF (VERY SMALL MACH NO)
C                                    C(12) WEIGHTED NORM Y
C                                    C(13) HMIN
C                                    C(14) HMAG
C                                    C(15) SCALE
C                                    C(16) HMAX
C                                    C(17) XTRIAL
```

```
C                                 C(18) HTRIAL                      DVEK3720
C                                 C(19) EST                         DVEK3730
C                                 C(20) PREVIOUS XEND               DVEK3740
C                                 C(21) FLAG FOR XEND               DVEK3750
C                                 C(22) NO OF SUCCESSFUL STEPS      DVEK3760
C                                 C(23) NO OF SUCCESSIVE FAILURES   DVEK3770
C                                 C(24) NO OF FCN EVALS             DVEK3780
C                                 IF C(1) = 4 OR 5, C(31),C(32),....  DVEK3790
C                                   C(N+30) ARE FLOOR VALUES        DVEK3800
   15 CONTINUE                                                      DVEK3810
C                               INITIAL ENTRY WITH OPTIONS (IND .EQ. DVEK3820
```

Appendix D

Material Property Data

It was necessary to perform a series of elastic modulus determination tests to characterize this adherent material. Slight variations in material properties can be evident in molding compounds even manufactured by the same supplier. In a separate, extensive study concerning material property data, Taggart reported the elastic modulus of SMC-25 to be 2.1×10^6 psi and the results shown in Table 4 are in close agreement.

Table 4

Specimen #	Modulus (PSI)
SPEC1	2.21×10^6
SPEC2	2.26×10^6
SPEC3	2.18×10^6

Plate 6 shows a typical test specimen used for modulus determination. The data for these tests may be found on the following pages.

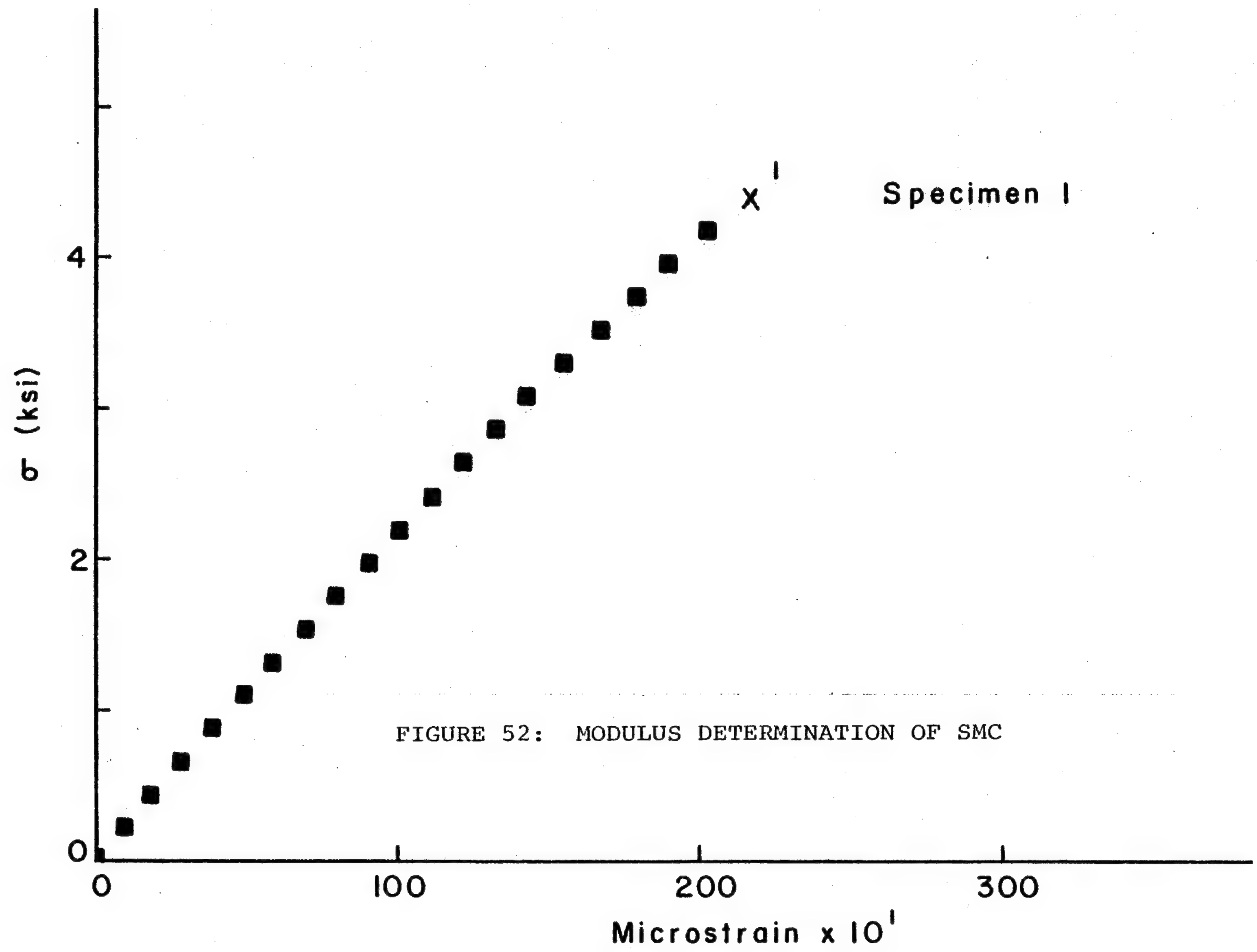

FIGURE 52: MODULUS DETERMINATION OF SMC

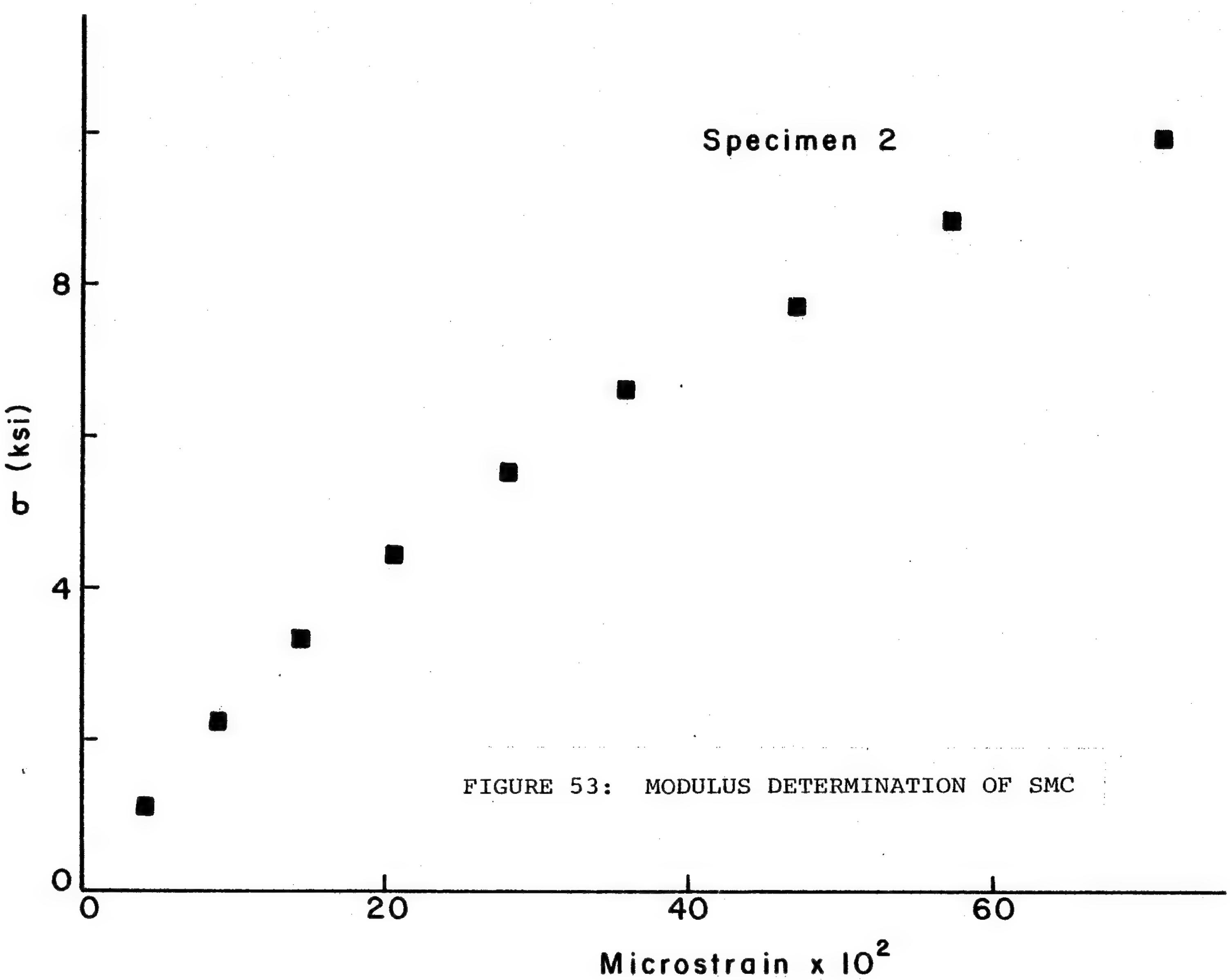

FIGURE 53: MODULUS DETERMINATION OF SMC

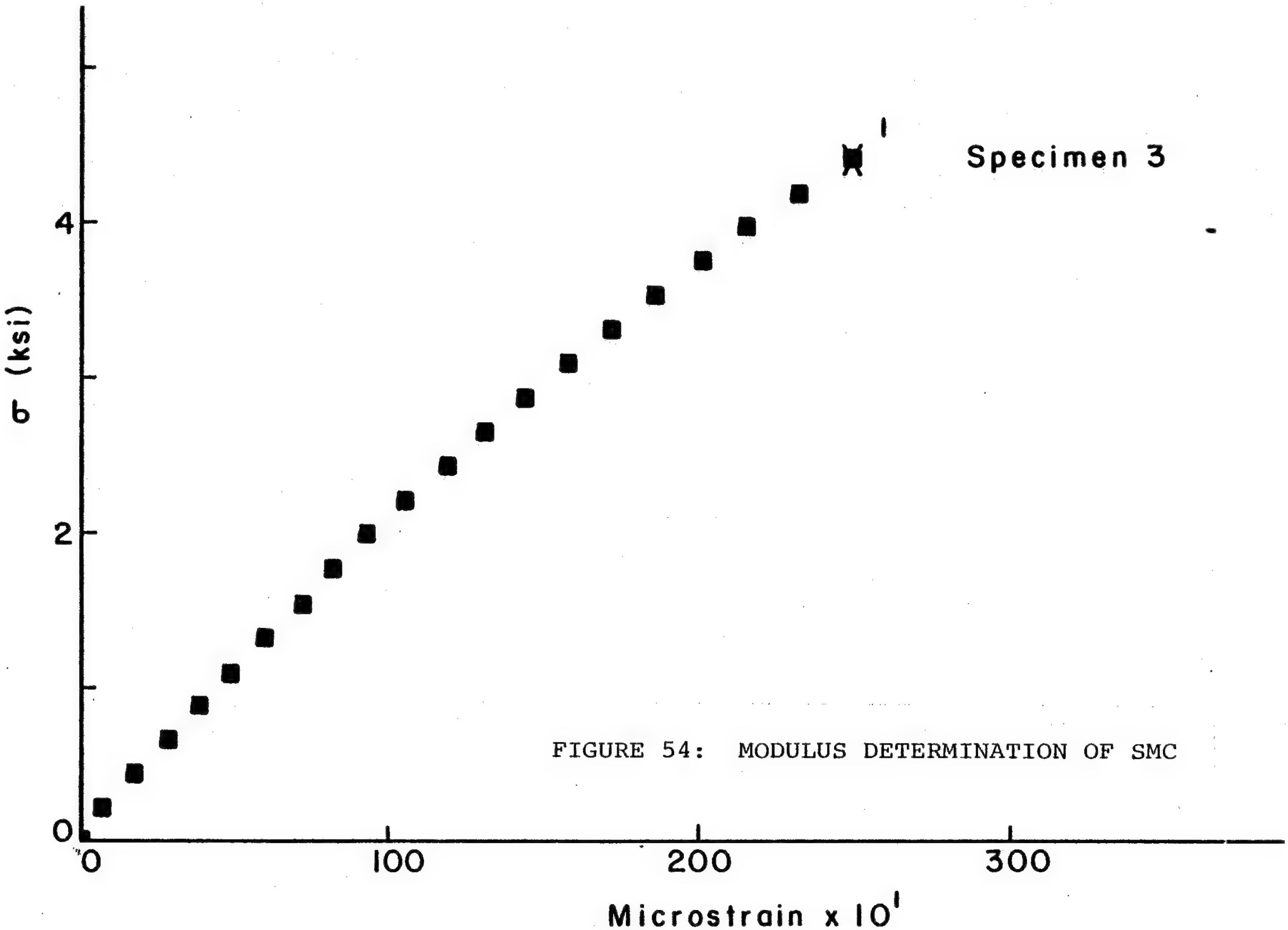

FIGURE 54: MODULUS DETERMINATION OF SMC

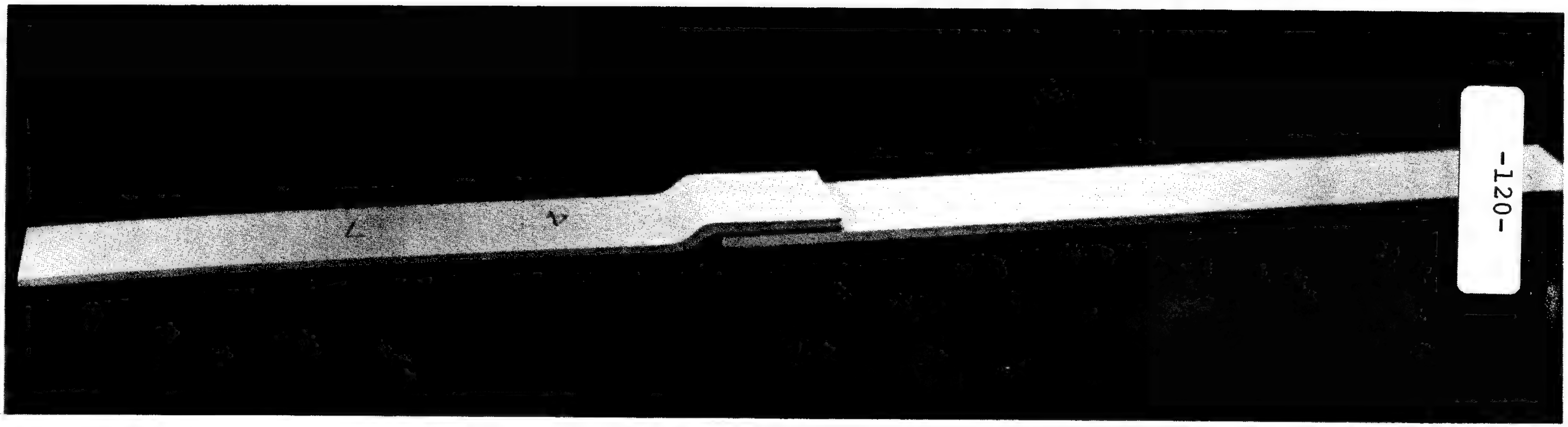

PLATE 1: Typical Test Specimen

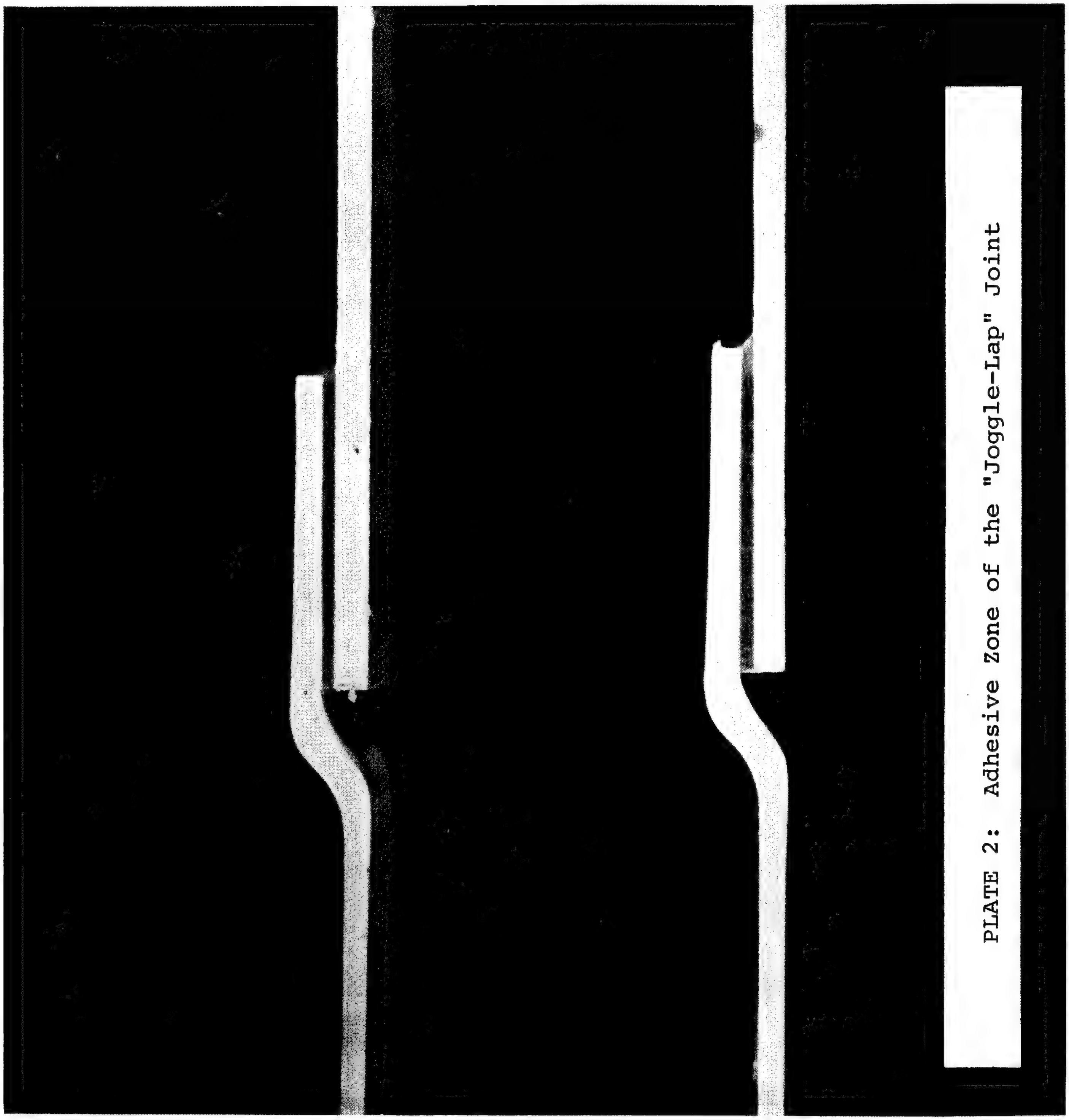

PLATE 2: Adhesive Zone of the "Joggle-Lap" Joint

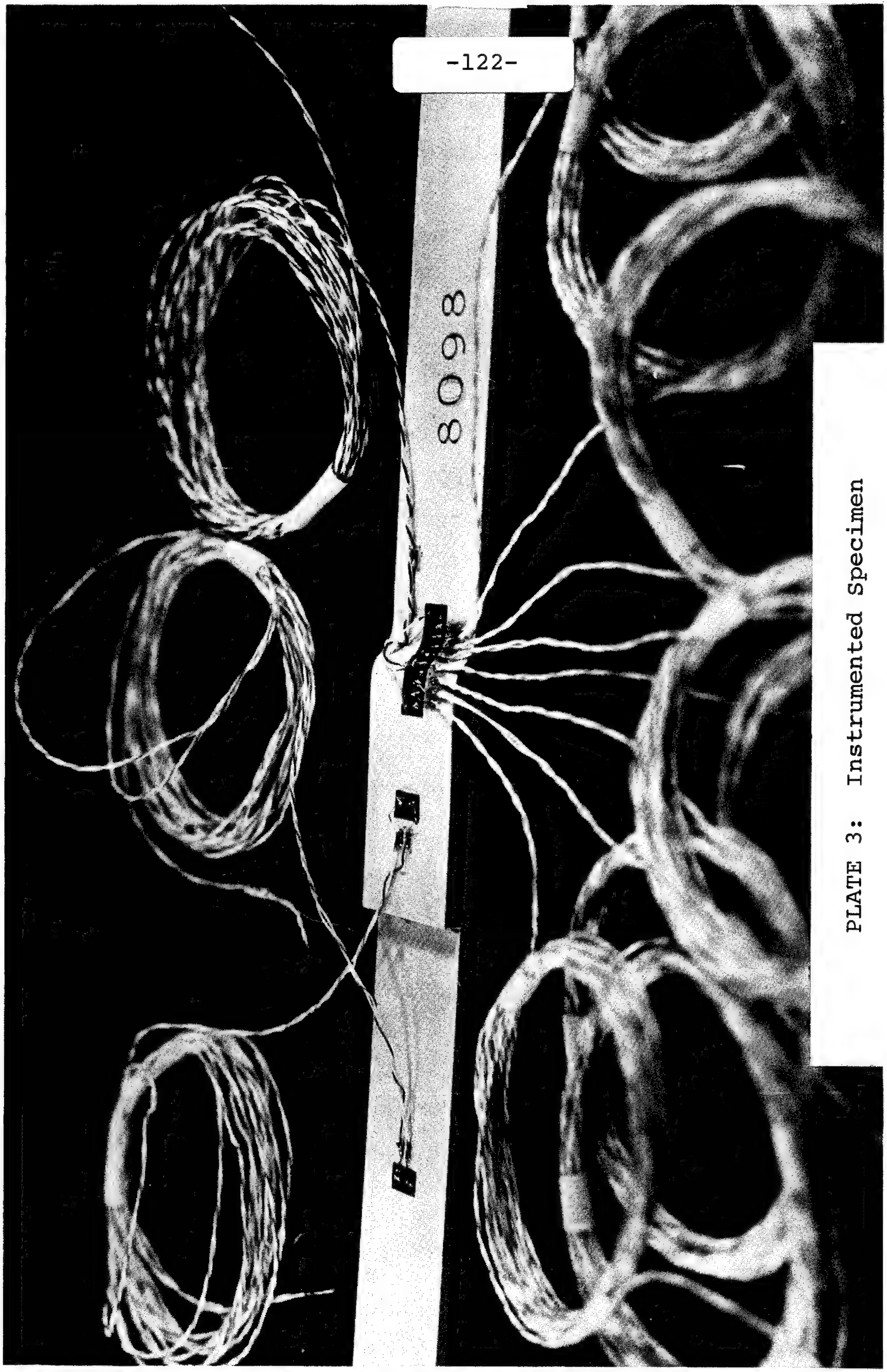

PLATE 3: Instrumented Specimen

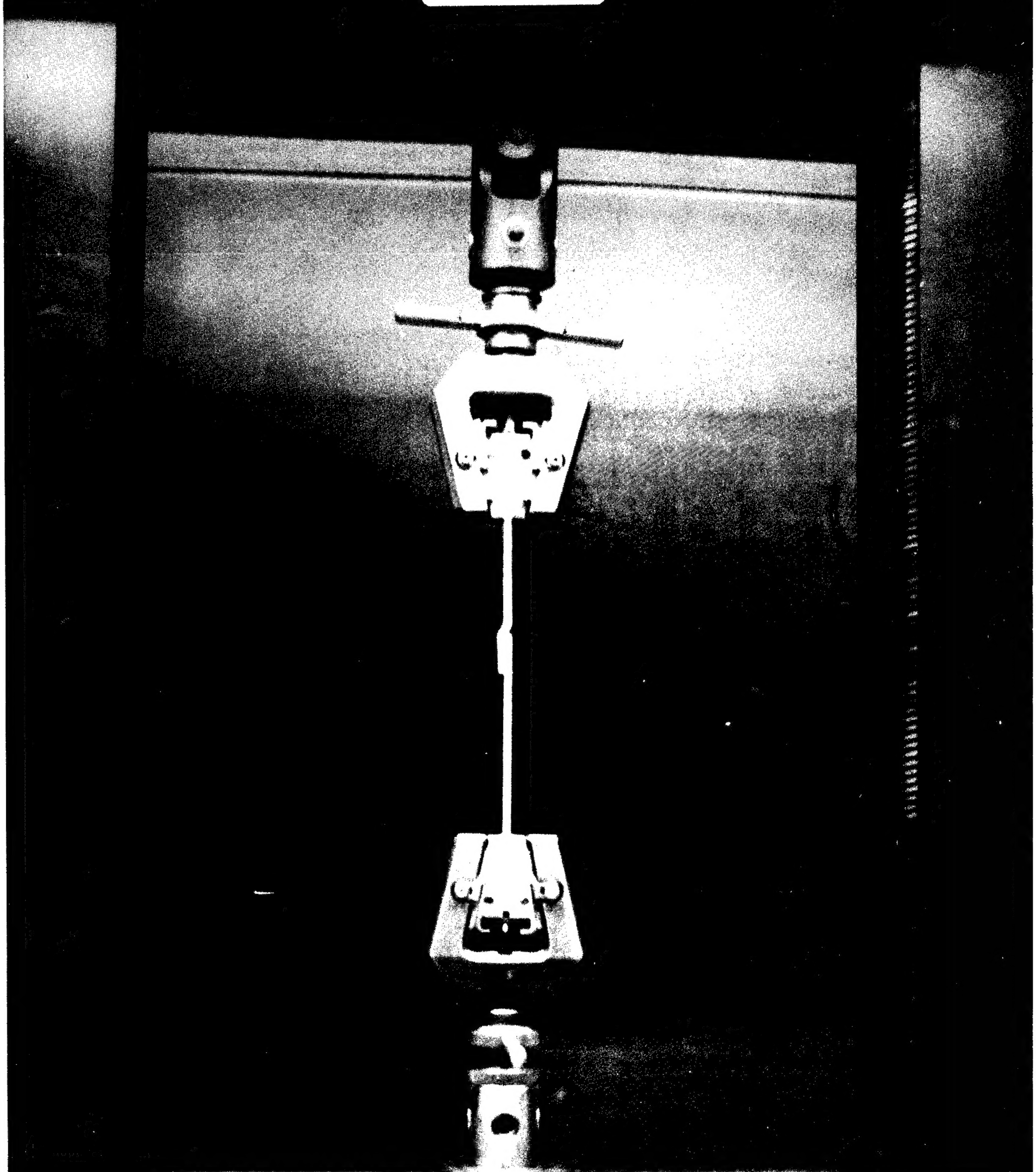

PLATE 4: "Joggle-Lap" Joint Subject to Tension

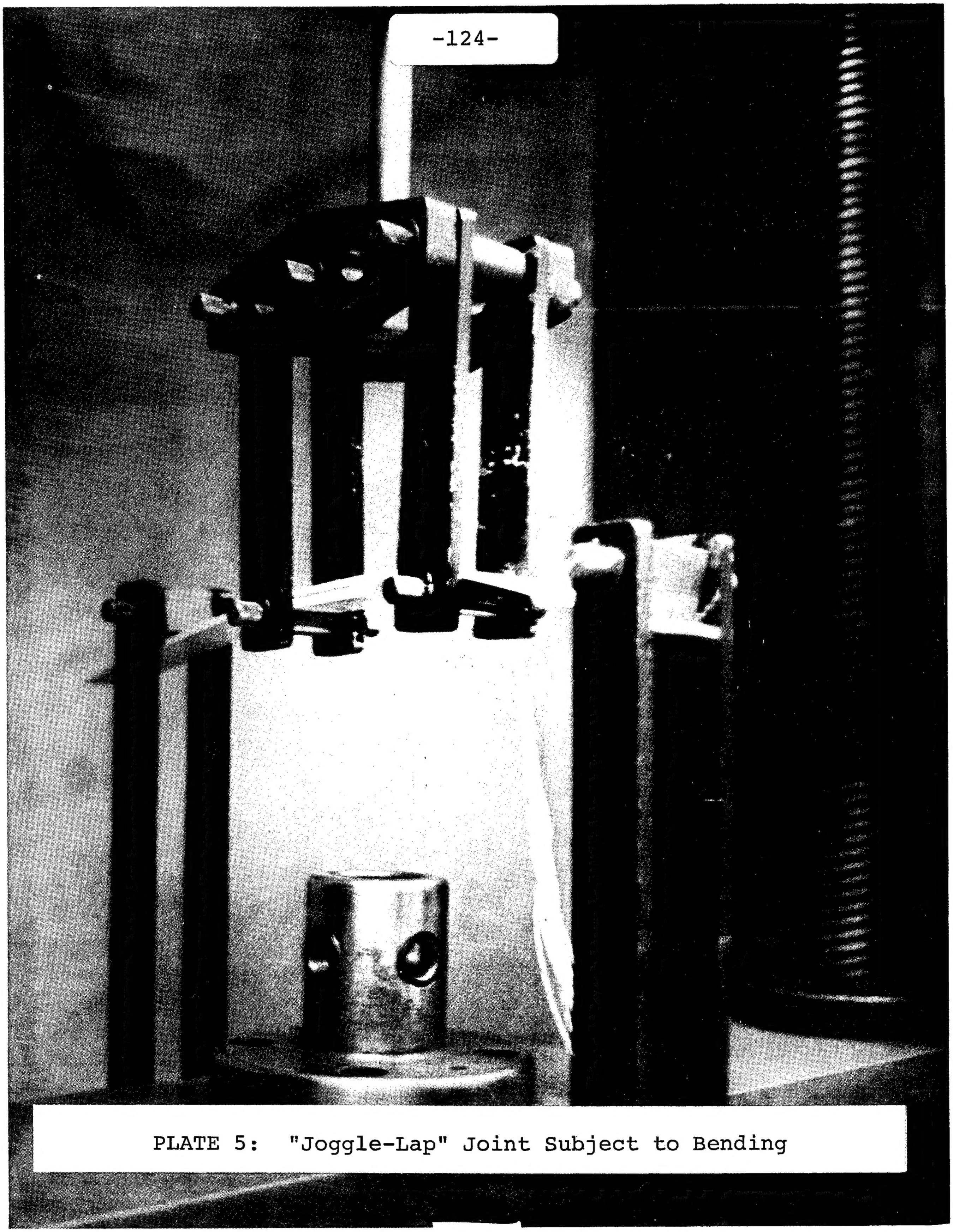

PLATE 5: "Joggle-Lap" Joint Subject to Bending

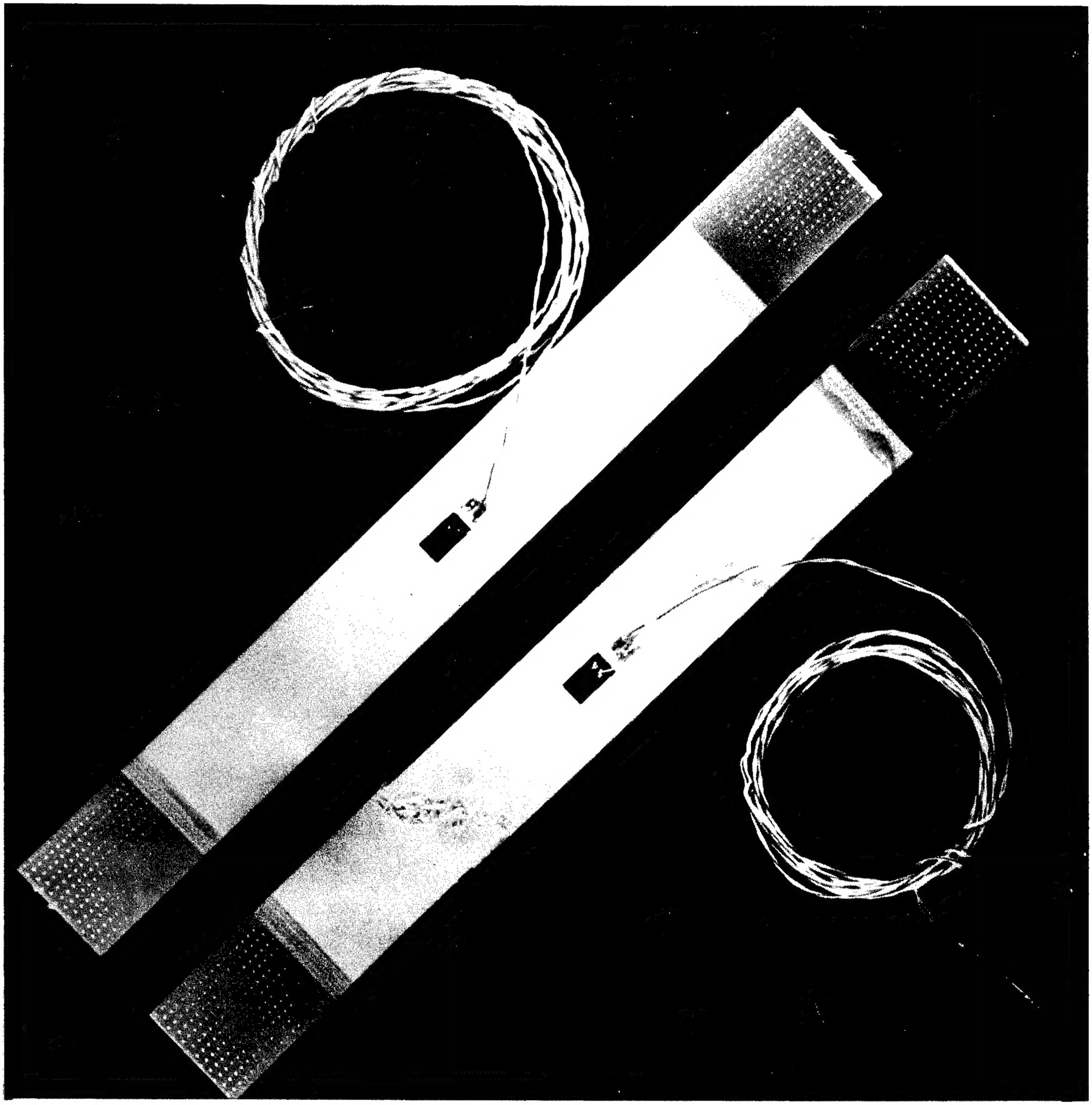

PLATE 6: Tensile Coupons for Modulus Determination

Curved Section (SEG3)

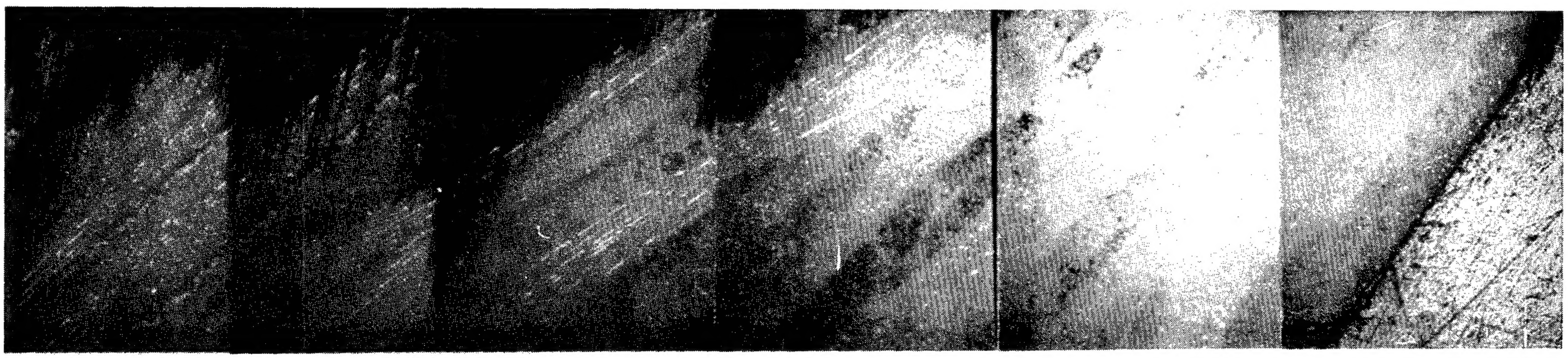

PLATE 7: Photomicrographs Showing Relative Fiber Content